Progression in Mental and Written Calculations

INTRODUCTION

This policy is a result of some rich discussions and much debate between Maths Leaders as part of a Calculation Progression Working Party. Collaborative work took place, including taking photographs to exemplify concrete approaches to introducing the mental calculation strategies and written methods with a range of resources commonly found in schools.

There is no specific resource that *must* be used as schools vary in what they have. What *is* important is that the resources used are consistent throughout an individual school so that pupils are familiar with the resources from year to year and can visualise those images when creating their own pictorial representations. This will support their conceptual understanding of the final abstract methods rather than using 'rote learnt' methods that pupils can both struggle to apply accurately and to know which methods to use when problem-solving.

Thank you to the following Maths Leaders and schools for their involvement in the Calculation Progression Working Party:

Aleha Begum	Goodmayes Primary
Caroline Dolman	John Bramston Primary School
Delvinder Singh (and previously Christian Atwell)	Atam Academy
Michelle Williams	Cranbrook Primary School
Vic York and Matt Wellsman	Seven Kings School
Paula Murray-Mower	Independent Mathematics Consultant www.pmm-maths.co.uk

ORGANISATION

This policy consists of:

Main section:

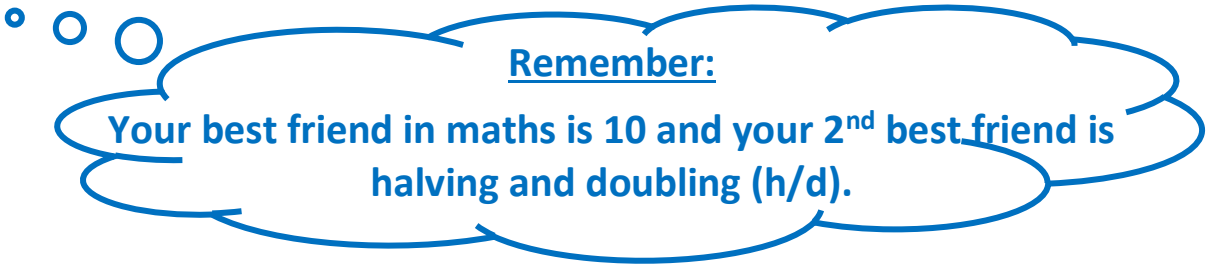
- **Progressive steps** through learning each mental strategy and written method, including the supportive/expected notation.
  This should be used in planning and over-rule any schemes used in school. Schemes may have appropriate support materials, which can be used, but the policy progression must be followed.

Additional information to meet expectations

- **Expectations of the Number Facts** (+ - x ÷) to be learnt in each year group
  Time must be dedicated to teach pupils how to learn these at school and at home.
- **Year Group Overview pages for each operation** that give the end of year expectations for efficient mental strategies and written methods, including the size of numbers involved, in-line with the expectations of the National Curriculum.
  These need to be referred to in planning and could be laminated and displayed in classrooms as a reference. They also provide previous year group expectations for those pupils working below year group expectation and an expectation of Greater Depth achievement. They should not be used to accelerate to the next year group content.
- **CPA Representations** (concrete/pictorial/abstract) for each operation in-line with the policy progression. There are fewer calculation resources shown as the pupils get older as the expectation is that they know their number facts and can apply these with their knowledge of place value. Those pupils working below year group expectations may still be using the resources for the calculations relevant to the year group at which they are working. However, other pupils should still be using resources to problem solve and stretch and challenge their thinking in other ways.

BEST FRIEND IN MATHS

Fluency in the recall of key number facts is a crucial aspect of expected standard and underpins much of the mathematics curriculum. Without this fluency, children are hindered in moving on in many areas of the curriculum, which rely on a good grasp of number. They will not meet expected standard and will struggle to cope with basic concepts and calculations.



10 is your Best Friend in Maths; Halving and doubling is your 2nd Best Friend (*not* the number 5). This forms the basis of the policy, using 10 and halving and doubling to support calculations. Children need to be taught to use their 'Best Friend and 2nd Best Friend' and reminded to look for ways that these may help them when calculating independently. Using your 'Best Friend and 2nd Best Friend' can then be applied in further work, making the links in learning e.g. fractions, decimals and percentages.

Using your Best Friend 10 includes the ability to:

- **Understand place value**
- **Partition and recombine**
- **Instantly recall facts *within* and *to* 10** (this enables children to 'jump' to the next multiple of 10 and also to add any number using partitioning and recombining).
- **x ÷ 10, 100 and 1000** and explain the effect

These facts need to be taught as appropriate for the age of the pupil; expectations of these are included in the policy.

NB:
As 100 is a multiple of 10 (or 'related to 100') it is used in the same way for larger numbers, e.g. jumping to the next multiple of 100 when subtracting.

Using your 2nd Best Friend halving and doubling includes the ability to:

- **Instantly recall doubles of all digits 1 to 9.**
- **Instantly recall halves of all even numbers to 18.**

These facts need to be taught as appropriate for the age of the pupil; expectations of these are included in the policy.
NB: Doubling and halving needs to be rehearsed from Reception to 6 to ensure ease and efficiency in mental and written calculations.

Non-Negotiables for KS2:

Year 3	Standard Column Addition ('carrying')
Year 4	Standard Column Decomposition for Subtraction ('repartitioning') Short Multiplication (including arrow jottings for mental)
Year 5	Short Division (including 'mental chunking') Long Multiplication
Year 6	Long Division

PROGRESSION FROM MENTAL TO WRITTEN METHODS OF ADDITION AND SUBTRACTION

BEFORE PROGRESSING THROUGH THE STAGES OF WRITTEN CALCULATION, THE FOLLOWING MENTAL SKILLS ARE CRUCIAL PREREQUISITES:

Remember: Best Friend in Maths is 10 and 2nd best friend is halving and doubling (h/d).

Learning

- It is crucial to know or be able to derive key number facts –

  Reception +/- totals to 5 (instant recall) then +/- facts within 5 and totals to 10

  Year 1 +/- totals to 10 (instant recall) then +/- facts within 10

  Year 2 +/- within at *least 10* (instant recall) and totals to 20 (instant recall)

  Year 3 +/- within 20 (instant recall or rapid mental calculation using known facts and place value)

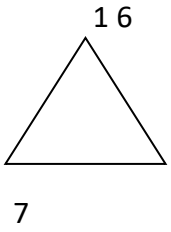
  Year 4 onwards 2-digit +/- 2-digit number (mental calculation using known facts and place value, possibly supported by a jotting NOT a vertical method)

  Year 5 and 6 Consolidation and practise of the previous key facts, including +/- numbers with 1 decimal place.

- Place value and partitioning MUST be clearly understood and explained using the appropriate mathematical vocabulary.

Teaching

- The number line must be modelled as an image to support mental subtraction from Year 1 to Year 6.
- Jottings, including the use of arrows prior to formal written methods, must be modelled as a clear image/strategy for mental calculation.
- Teach the 'three related numbers' so that links between the two operations are recognised, e.g.



7 + 9 = 16
9 + 7 = 16
16 – 9 = 7
16 – 7 = 9

Always present calculations horizontally in order to consider mental calculations first.

Always think:
1. *Can I do it mentally?*
2. *Can I do it with a jotting?*
3. *Do I need a written method?*

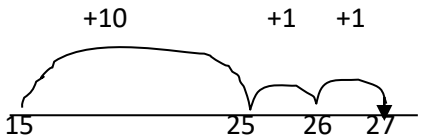
Progression in Calculations

ADDITION

1. COUNTING ON IN 1s

2. COUNTING ON IN 10s AND 1s

15 + 12 = 27



Year 1

Model with a bead bar, using a 'tens-catcher' to model counting on in 10s)

Number line Teaching Points:

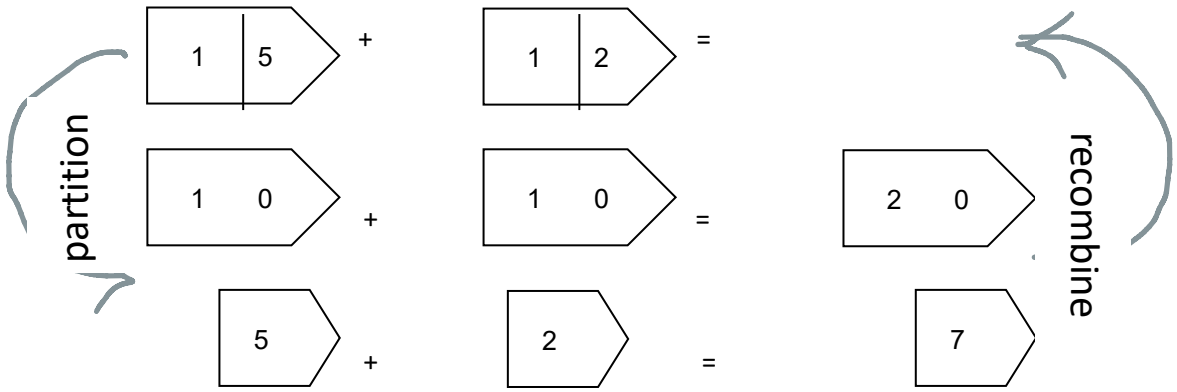
- Always work with numbers reading from left to right (smallest to largest), whatever the operation of the calculation.
- Numbers ('landmarks') are written below the line.
- MUST show an arrow at the end of the 'jumps' to show direction.
- Size of the 'jumps' are written above the 'jumps'.

AS SOON AS CHILDREN ARE COMPLETELY SECURE IN THEIR UNDERSTANDING OF PLACE VALUE AND THEY USE THE APPROPRIATE VOCABULARY TO RECOGNISE AND EXPLAIN 24 IS "2 TENS AND 4 ONES" THEN MOVE ON TO CALCULATING BY:

3. PARTITIONING AND RECOMBINING

Year 2

15 + 12 = 27



Model and practise with place value arrow cards, using known facts and place value to calculate each step.

Moving onto recording as:

34 + 23 = 57
30 + 20 = 50
4 + 3 = 7

It is important to model the use of the arrow combination in calculation jottings. These can later be applied to other jottings and concepts.

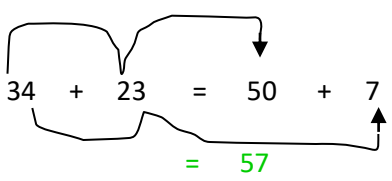
Express this as:
"30 add 20 equals 50"
AND as:
"3 tens add 2 tens equals 5 tens or 50"

Write equals under equals rather than write across the page. This organised layout makes the question and final answer clear.

As well as place value arrow cards, also model this on a bead bar and practise on 100-beadstrings, showing the 'collection' of 10s and then the ones. i.e "3 tens and 2 tens makes 5 tens, which is 50. Then 4 and 3 makes 7 ones. Altogether we can see 5 tens and 7 ones, which is 57." Check by counting in tens and ones along the 100 bead bar.

Progression in Calculations

Moving onto a more efficient recording:



"chains"

Years 2 & 3 onwards and mentally through KS2 for addition of 2-digit numbers

48 + 35 = 70 + 13
= 83

Year 2 EXS TAF: Add any 2 two-digit numbers using an *efficient strategy*, explaining their method verbally, in pictures or using apparatus (e.g. 48 + 35).

Stop using the 'chain' as soon as the concept of 'chaining up the tens and then the ones' is grasped. The chain acts as a reminder model and is not needed on the jotting once the child is secure with the strategy. Eventually, the jotting won't be needed at all as the calculation will be done entirely mentally (expectation by Year 4).

PROGRESSIVE STEPS FOR EACH STAGE OF PARTITIONING AND RECOMBINING:

- Not crossing tens, e.g. 24 + 35
- Crossing tens, e.g. 24 + 17
- Crossing hundreds only, e.g. 56 + 61
- Crossing tens and hundreds, e.g. 76 + 85

CONTINUE USING THIS NOTATION/JOTTING (WITH/WITHOUT CHAIN ACCORDING TO NEED) WHEN EXTENDING TO MENTAL ADDITION OF:

- pairs of 3-digit numbers, if appropriate (Year 3⁺)
- pairs of decimals when decimal place value knowledge is secure (Year 5⁺)

When crossing ones, express this as:
"3 tenths add 8 tenths equals 11 tenths, which is 1 whole one and 1 tenth, which is 1.1"

ONLY MOVE TO THE NEXT STAGE IF THE NUMBER FACTS WITHIN 20 ARE KNOWN AND TU + TU CAN BE CARRIED OUT MENTALLY (possibly supported by a jotting)

If the calculation should be possible mentally then do not give it to practise vertical calculation, e.g. TU + TU should not be calculated vertically. Consider use of numbers carefully, including the progressive steps within a stage. When secure, teach children to choose the most appropriate method and include calculations that can be carried out mentally/with a jotting so that children 'spot' them.

Remember: ALWAYS PRESENT CALCULATIONS HORIZONTALLY IN ORDER TO CONSIDER MENTAL CALCULATIONS FIRST.

Year 3

4. ADDING THE MOST SIGNIFICANT DIGITS FIRST

123 + 145 = 268

```
  1 2 3
+ 1 4 5
  2 0 0
    6 0
      8
  2 6 8
```

Working from left to right:
"1 hundred + 1 hundred is 200
2 tens + 4 tens = 6 tens, which is 60 or 20 + 40 is 60
3 + 5 is 8"

'Read' the answer from left to right, using knowledge of place value:
"two hundred and sixty-eight" NOT adding up columns for the final answer.

If a calculation should be possible mentally then do not give it to practise vertical calculation, e.g. TU + TU should not be calculated vertically. Consider use of numbers carefully, including the differentiation steps within a stage. When secure, teach children to choose the most appropriate method and include calculations that can be carried out mentally/with a jotting so that children 'spot' them.

PROGRESSIVE STEPS FOR EACH STAGE OF ADDING THE MOST SIGNIFICANT DIGITS FIRST:

- Not crossing tens e.g. 132 + 146
- Crossing tens e.g. 127 + 154
- Crossing hundreds only e.g. 178 + 131
- Crossing tens and hundreds e.g. 175 + 147

5. ADDING THE LEAST SIGNIFICANT DIGITS FIRST

Year 3

323 + 245 = 568

3 2 3
+ 2 4 5
8
6 0
5 0 0
5 6 8

Working from right to left:
"3 + 5 is 8
2 tens + 4 tens = 6 tens, which is 60 or 20 + 40 is 60
3 hundred + 2 hundred is 500"

'Read' the answer from left to right in an upwards direction, using knowledge of place value:
"five hundred and sixty-eight" (still NOT adding up columns for the final answer)

PROGRESSIVE STEPS FOR EACH STAGE OF ADDING THE LEAST SIGNIFICANT DIGITS FIRST:

- Not crossing tens e.g. 432 + 246 Consider mental strategy of partitioning and recombining first
- Crossing tens e.g. 527 + 354
- Crossing hundreds only e.g. 378 + 431
- Crossing tens and hundreds e.g. 875 + 247

ENSURE THE APPROPRIATE LANGUAGE OF PLACE VALUE IS USED, UNDERSTOOD AND THE VALUE OF EACH DIGIT IS EXPRESSED.* It is not 2 + 4, it is 20 + 40 is 60 or 2 tens + 4 tens is 6 tens, which is 60*

6. COLUMNAR ADDITION ('carrying')

Year 3 onwards

427 + 254 = 681

4 2 7
+ 2 5 4
6 8 1
1

Working from right to left:
"7 + 4 is 11. Partition the 11 into 10 and 1, 'carry' the ten into the tens column, writing it as 1, below the line, to represent one ten." *It is NOT "carry the 1"

"1 ten added to 2 tens and 5 tens is 8 tens" or "10 + 20 is 30; plus 50 is 80". Write this as 8 in the tens column to represent 8 tens or 80.

"4 hundred + 2 hundred is 600". Write this as 6 in the hundreds column to represent 600.

*Digits must be expressed as their appropriate values, NOT as single-digits i.e. *20* or *2 tens* NOT '2'.

PROGRESSIVE STEPS FOR THE COLUMNAR ADDITION:

- Not crossing tens e.g. 432 + 246
- Crossing tens e.g. 527 + 354
- Crossing hundreds only e.g. 378 + 431
- Crossing tens and hundreds e.g. 875 + 247

Year 3: 3-digit numbers
Year 4: 4-digit numbers
Years 5 & 6: 5-digit numbers, including decimals

IT IS IMPORTANT TO PROGRESS THROUGH EACH STEP FOR EACH STAGE OF CALCULATING ADDITION, REVERTING BACK TO THE FIRST STEP EACH TIME A NEW STAGE BEGINS.

LIKEWISE, WHEN EXTENDING TO ADDITION OF DECIMALS, REVERT BACK THROUGH THE STAGES OF PROGRESSION FROM 'CHAIN' TO ADDING MOST THEN LEAST SIGNIFICANT DIGITS BEFORE THE STANDARD WRITTEN METHOD.

USE THE FOLLOWING STEPS THROUGH EACH OF THOSE STAGES OF PROGRESSION FOR ADDING DECIMALS (Year 5+):

Jottings show partitioning & recombining ('chain') stage of progression.
Number/progressive steps are the same regardless of the stage of progression.

Years 5 & 6

One decimal place (1 d.p.)

- Not crossing ones (units) e.g. 1.3 + 1.4

- Crossing ones e.g. 3.5 + 1.7 = 5.2

  3.5 + 1.7 = 4 + 12/10
  = 4 + 1.2
  = 5.2

When adding decimal fractions, express as fractions, i.e. Say 'tenths' not 'point …', i.e. Say "3 tenths add 4 tenths is 7 tenths" not "point 3 + point 4 = point 7." Expressing the fractional size is more meaningful.
When crossing ones, express this as:
"5 tenths add 7 tenths equals 12 tenths, which is 1 whole one and 2 tenths, which is 1.2"
Remind pupils that it is not 0.12 because it that is less than a whole one.

Note: We do not say "zero point twelve". It is "zero point one two".

Two decimal places (2 d.p.)

- Not crossing ones (units) or tenths e.g. 1.14 + 5.35 (as for 1d.p. above)

- Crossing tenths only e.g. 1.28 + 2.34 = 3.62

  1.28 + 2.34 = 3 + 62/100
  = 3.62

The assumption is that, if adding decimals, the pupils are able to add pairs of 2-digit numbers mentally. Also, that the value of hundredths in a decimal fraction with 2 decimal places is FULLY understood. Use this knowledge to add hundredths:

Say, " 28 hundredths add 34 hundredths is 62 hundredths, which is 0.62".

- Crossing ones only, e.g. 2.42 + 1.84 = 4.26

  2.42 + 1.84 = 3 + 126/100
  = 3 + 1 + 26/100
  = 4.26

Years 5 & 6

Remember: ALWAYS PRESENT CALCULATIONS HORIZONTALLY IN ORDER TO CONSIDER MENTAL CALCULATIONS FIRST.

- Crossing ones and tenths e.g. 1.75 + 4.47 = 6.22

  1.75 + 4.47 = 5 + 122/100
  = 5 + 1 + 22/100
  = 6.22

SUBTRACTION

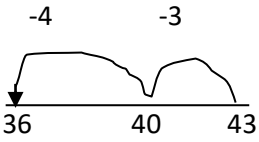
It is important that children understand subtraction as 'take away' and as 'finding the difference' and to be able to interpret the context when faced with a subtraction calculation or problem. Even with 'finding the difference', a number may still be taken away and then we count on to find how much is left (the difference).

1. **COUNTING BACK IN 1s**

2. **COUNTING BACK (for a large difference,** i.e when subtracting a small number)

43 – 7 = 36

Year 1

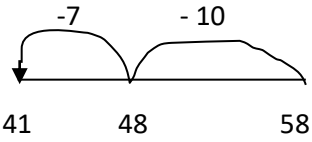


Using known facts and place value, partition 7 into 3 + 4 to land on the multiple of 10 and continue counting back.

3. **COUNTING BACK IN 10s and 1s (for a larger difference up to 20)**

Year 1

58 – 17 = 41



Using known facts and place value, partition 17 into 10 + 7 to count back 10 then back 7 more.

Model this using the tens-catcher to count back 10 to highlight that the tens digit changes and the ones remain the same when subtracting 10.

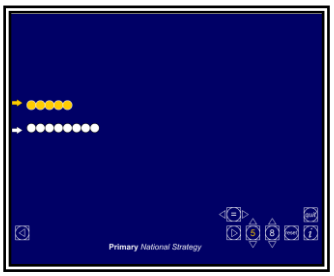
When dealing with larger numbers (beyond subtraction of 'teens' numbers, where place value and known facts can be used quickly) **then COUNTING ON TO FIND THE DIFFERENCE is then used as the most RELIABLE and EFFICIENT strategy.**

1. **COUNTING ON IN 1s**

Year 1

2. **COUNTING ON (initially for a small difference** i.e. 2 numbers which are close together, subtracting a large number) **COUNTING ON TO FIND THE DIFFERENCE**

NB: It is important to spend a lot of time (initially in Year 1) on the concept of 'difference'. This can be demonstrated in using concrete resources by first comparing children's features (hair colour, glasses, cardigan or jumper, etc.) then moving on to ordering children's heights and then towers of cubes (in single colours) and other counting equipment.
Use the language *"How much more…?"*

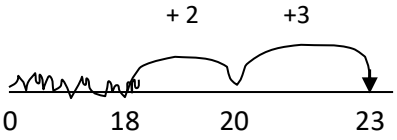


Moving onto a middle step between the concrete and the pictorial: use the Difference ITP to demonstrate 'finding the difference'.

Year 1

After lots of concrete practice and pictorial recording, moving onto the number line jotting:

23 – 18 = 5



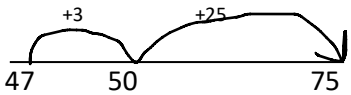
Initially, model the 'taking away' of the smallest number by 'scribbling out' the numbers from zero to that number to demonstrate that we are finding the difference; *how much more…?*

ALWAYS use 'landmarks' that are multiples of 10, i.e. land on the next multiple of 10 (or 100 when doing calculations with larger numbers) using known facts to work out the size of the jump. Write the landmarks on first then do each jump, labelling the size above the jump each time (i.e. not at the end).

It is vital that children have an idea of the position of numbers in the number system to be able to recognise when to count on, when numbers are close together or near a multiple of 10, 100 or 1000. The hundred square is NOT a suitable model for this. The bead bar, which is a linear model, is far clearer.

Year 2 EXS TAF: Subtract any 2 two-digit numbers using an *efficient strategy*, explaining their method verbally, in pictures or using apparatus (e.g. 72 – 17).
Two jumps (to the next ten and on to the largest number) would be most *efficient.*

75 – 47 = 28



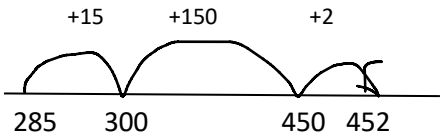
Years 2 & 3 and mentally through KS2 for subtraction of 2-digit numbers

Progression in Calculations

COUNTING UP ON A NUMBER LINE SHOULD BE USED AND EXTENDED THROUGHOUT THE SCHOOL AS A MENTAL STRATEGY FOR SUBTRACTION (with a number line support if needed).

- larger numbers

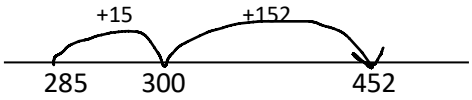
452 – 285 = 167



Again, ALWAYS use 'landmarks' that are multiples of 10, i.e. land on the next multiple of 10 (or 100 when doing calculations with larger numbers) using known facts to work out the size of the jump. Write the landmarks on first then do each jump, labelling the size above the jump each time (i.e. not at the end).

- larger numbers with more efficient recording

452 – 285 = 167

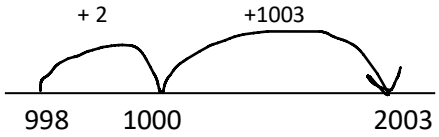


Initially with jottings (number line) then as a mental strategy through KS2 where possible

The expectation is that this will eventually be carried out MENTALLY in just two steps ('jumps').

- larger numbers but where the numbers are close to multiple of 100 or 1000. This should then be carried out MENTALLY.

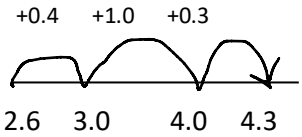
2003 – 998 = 1005



Remember: ALWAYS PRESENT CALCULATIONS HORIZONTALLY IN ORDER TO CONSIDER MENTAL CALCULATIONS FIRST.

- decimals

4.3 – 2.6 = 1.7

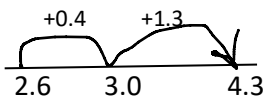


Revert back to this stage (counting up on a number line) when using decimals, even if at a higher stage of progression for subtraction of whole numbers, as it helps clarify the size of decimals and gives a clear visual image to emphasise the place value involved.

Extend to carrying out with a minimum of 2 jumps' (more efficient recording)

Years 5 & 6

4.3 – 2.6 = 1.7



If the calculation should be possible mentally then do not give it to practise vertical calculation, e.g. TU - TU should not be calculated vertically. Consider use of numbers carefully, including the progressive steps within a stage. When secure, teach children to choose the most appropriate method and include calculations that can be carried out mentally/with a jotting so that children 'spot' them.

Progression in Calculations

NB: THE FOLLOWING METHOD HAS NO PROGRESSIVE LINK FROM ANY PREVIOUS METHOD OF SUBTRACTION BUT STANDS ALONE AS A SEPARATE METHOD:

3. SUBTRACTION BY EXPANDED DECOMPOSITION (from the end of Year 3 and only if clear understanding of place value and correct vocabulary usage during the process of calculation; as well as an excellent recall of required addition and subtraction facts for Year 3+)

Year 3

Remember: ALWAYS PRESENT CALCULATIONS HORIZONTALLY IN ORDER TO CONSIDER MENTAL CALCULATIONS FIRST.

- Subtracting with no repartitioning needed:

345 – 123 = 222

```
  300  +  40  +  5
- (100  +  20  +  3)
-------------------
  200  +  20  +  2
-------------------
```

Partitioning each number and working from right to left, subtracting the bottom number from the top. Express each part as its value represented, i.e ."40 – 20".

- Moving onto subtracting with repartitioning of tens only:

Practise **repartitioning** a number before teaching this stage of progression, e.g. lesson starters, partitioning in different ways, e.g. 57 = 50 + 7; 57 = 40 + 17

252 – 114 = 138

```
  200  +  50  +  2
- (100  +  10  +  4)
-------------------
                  ?
-------------------
```

⇨

```
                40        12
  200  +  ~~50~~  +  ~~2~~
- (100  +  10  +  4)
-------------------
  100  +  30  +  8
-------------------
```

Year 3

Again, partitioning each number and working from right to left, subtracting the bottom number from the top. Where the subtraction is not possible i.e. 2 – 4 can't be done (without giving a negative number) the next value is "REPARTITIONED". So, "repartition 50 + 2 into 40 + 12". It is important to cross out the whole number and replace completely. Do NOT put a 'one in the air'! (It is not a 1, it is a 10.) Then repeat the subtraction process, this time "12 – 4 = 8" and "40 – 10 =30"

PROGRESSIVE STEPS FOR EACH STAGE OF EXPANDED DECOMPOSITION:

- Not repartitioning tens e.g. 458 - 436
- Repartitioning tens e.g. 547 - 328
- Repartitioning hundreds only e.g. 635 - 143
- Repartitioning tens and hundreds e.g. 725 - 437

Remember, always present calculations horizontally and consider mental/jottings before using a vertical written method. Do not give pupils calculations they can do with a mental strategy if you want them to practise a vertical method. Encourage pupils to look out for alternative strategies and allow them to use these.

4. SUBTRACTION BY STANDARD DECOMPOSITION

546 – 328 = 218

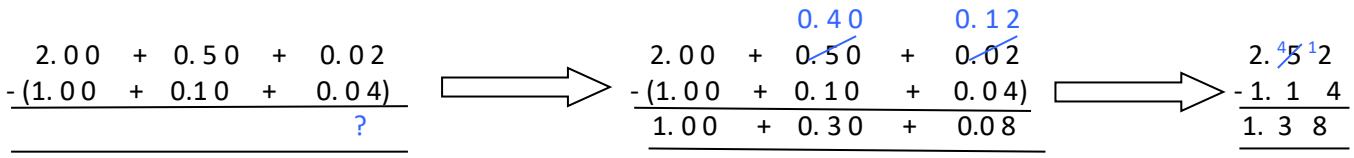
```
  5 3 ~~4~~ 1 6
- 3  2  8
---------
  2  1  8
---------
```

It is still vital that the correct language of place value is used. The tens are REPARTITIONED (not "'borrow' a 1" and it is not "3 takeaway 1" but "300 takeaway/subtract/ minus 100."

By Year 4

- Extend to decimals: **Years 5 & 6**

2.52 – 1.14 = 1.38



PROGRESSION FROM MENTAL TO WRITTEN METHODS OF MULTIPLICATION AND DIVISION

Fluency in the recall of key number facts is a crucial aspect of expected standard and underpins much of the mathematics curriculum. Without this fluency, children are hindered in moving on in many areas of the curriculum, which rely on a good grasp of number. They will not meet expected standard and will struggle to cope with basic assessments.

AS WITH ADDITION AND SUBTRACTION, BEFORE PROGRESSING THROUGH THE STAGES OF CALCULATION, THERE ARE KEY PREREQUISITES SKILLS AND STRATEGIES TO BUILD ON:

Learning

- It is crucial to know or be able to derive key number facts –

  Understand and use doubling and halving

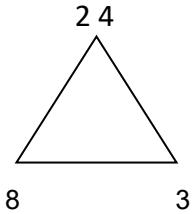
  ×/÷ 10 (as moving a place to the left/right NOT "add a zero" etc!!)

- Place value and partitioning MUST be clearly understood and explained using the appropriate mathematical vocabulary.

Teaching

- The number line, the use of arrays and arrow jottings must be modelled as images to support calculation from Year 1 to Year 6.
- Jottings must be modelled as a clear image/strategy for mental calculation.
- Teach the 'three related numbers' so that links between the two operations are recognised,

  e.g.



8 x 3 = 24
3 x 8 = 24
24 ÷ 8 = 3
24 ÷ 3 = 8

Always present calculations horizontally in order to consider mental calculations first.

Always think:
1. *Can I do it mentally?*
2. *Can I do it with a jotting?*
3. *Do I need a written method (vertical layout)?*

If the calculation should be possible mentally then do not give it to practise vertical calculation, e.g. 23 x 15 should not be calculated vertically. Consider use of numbers carefully. When secure, teach children to choose the most appropriate method and include calculations that can be carried out mentally/with a jotting so that children 'spot' them.

Progression in Calculations

MENTAL STRATEGIES

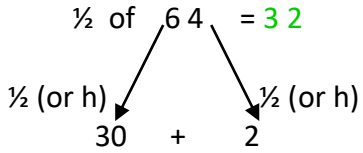
Learning times-tables, while making calculating more efficient, may not be possible for EVERY child as not ALL children will be able to learn ALL multiplication facts. However, strategies to calculate the facts not yet recalled ARE essential:

× 2	double	÷ 2	halve
× 4	double-double	÷ 4	half and half again
× 8	double-double-double	÷ 8	half, half and half again

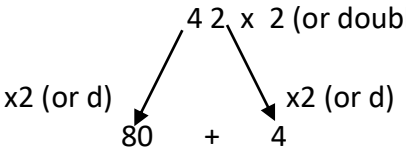
Years 1 & 2 onwards

Year 3 onwards through KS2

Model jottings for halving and doubling and use known facts and place value:



"Half of 6 tens or half of 60 is 3 tens or 30"
"Half of 4 is 2."

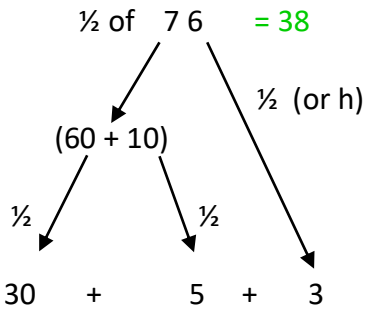


Year 2 onwards

"Double 4 tens is 8 tens or 80"
"Double 2 is 2."

Where the number of tens (or hundreds) is odd and the fact unknown, use known facts to derive the new fact:

e.g.



Year 3 onwards, including larger numbers

Where half of an odd number of tens is unknown, partition into an even number of tens plus 10, e.g. partition 70 into 60 + 10 and halve the two parts separately.

Stop using the arrows as soon as the concept is grasped. The arrows act as a reminder model and are not needed once the child is secure with the strategy. They may then just jot down 'holding numbers' – numbers they can't hold in their head – to help them 'see' the answer.

Using best friend (10 and second-best friend (h/d), teach and learn that:

Year 3 onwards: × 5 ½ of × 10 (× 10 then halve it)

Years 5 & 6:

× 50 ½ of × 100 (× 100 then halve it)

× 25 ¼ of × 100 (× 100 then ½ and ½ again)

× 12 × 10 plus × 2 (double)

× 15 × 10 plus ½ of × 10

Introduce by modelling halving groups of a number using the **Multiarray ITP**.
Use a jotting at first (see below) then write down the first answer as a 'holding number' and calculate mentally.

Model that x 5 is half of x 10 is using the multiarray ITP. If you halve 10 groups of a number, you get 5 groups of that number,

e.g. 5 x 28 = 140

½

28 x 10 = 280

e.g.

Year 3 onwards

3 2 x 5 = 160

↓ x10

320

↓ h

160

Years 5 & 6

2 4 x 5 0 = 1200

↓ x100

2400

↓ h

1200

2 8 x 2 5 = 700

↓ x100

2800

↓ h

1400

↓ h

700

NB Strategies and jottings are the same for dividing 2-digit numbers by 5, 50 and 25 but in reverse:

Year 3 onwards

÷ 5 ÷ 10 and double the number

÷ 50 ÷ 100 and double the number

Years 5 & 6

÷ 25 ÷ 100 and multiply by 4 (double, double)

More mental strategies to be taught when multiplying 2-digit numbers, again using best friend (10 and second-best friend (h/d):

Years 5 & 6

× 12 × 10 plus × 2 (double)

× 15 × 10 plus ½ of × 10

e.g.

2 4 x 1 2 = 2 8 8

x 10 → 240 ; x 2 (or d) → 48

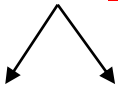
240 + 48

3 2 x 1 5 = 4 8 0

x 10 → 320 ; x 5 (by halving the x 10 answer) → 160

320 + 160

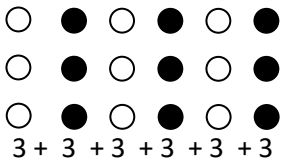
It is important that the arrows are joined in these jottings to show that the whole number is multiplied and not its partitioned parts.



For all of these arrow jottings. stop using the arrows as soon as the mental concept is grasped. The arrows act as a reminder model and are not needed once the child is secure with the strategy. They may then just jot down 'holding numbers' – numbers they can't hold in their head – to help them 'see' the answer.

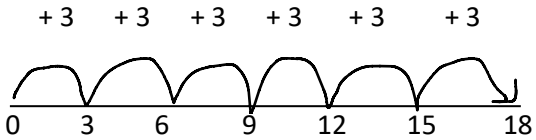
MULTIPLICATION

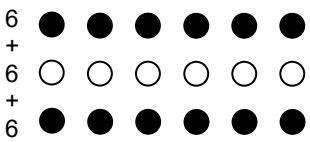
1. ARRAYS



shows 3 × 6 = 18

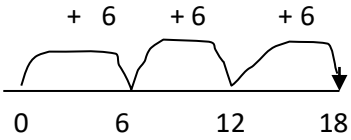
"3 multiplied by 5" is NOT "3 lots of 5" but "3 grouped 5 times". The 3 is the times-table, 5 is 'happening to it'; the 3 is being operated on,
e.g. for "3 multiplied by . . .", we start with the 3 and repeat it 5 times.



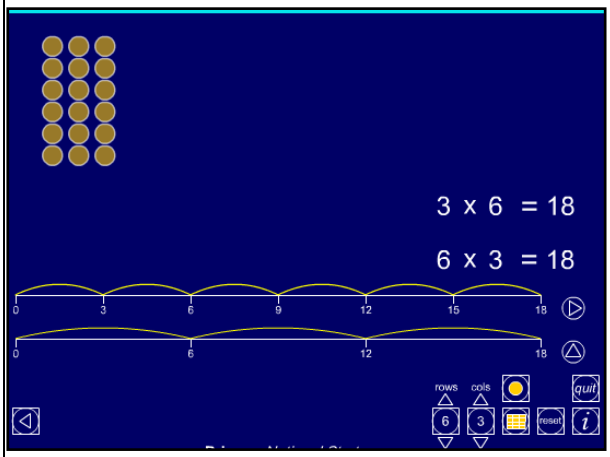


shows 6 × 3 = 18

"5 multiplied by 3" or "5 grouped 3 times". This is the 5 times table. The 5 is being operated on. It is *not* "5 lots of 3"



Years 2 & 3 for unknown times tables only



Use **Multiplication facts ITP** to model the link between the array and repeated addition on a number line.

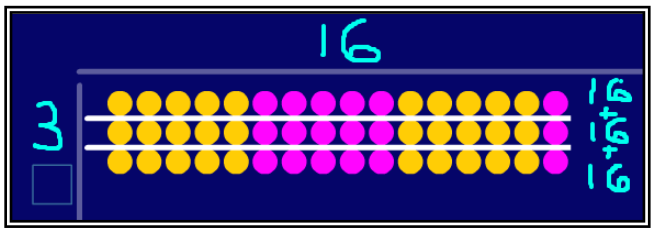
Concrete representations are vital:
Use wrapping paper arrays/egg boxes and multilink cubes etc. "Show me 3 x 4, 6 x 3" etc. Draw the matching number line.

2. PARTITIONING

Use Multiarray ITP to model how 16 is multiplied three times: 3 rows (groups) of 16 = 16 + 16 + 16

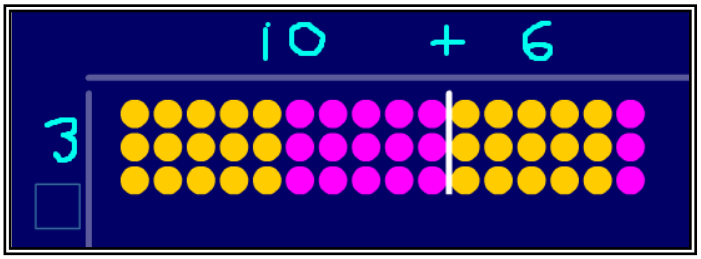
16 x 3 is "16 grouped 3 times"

partition



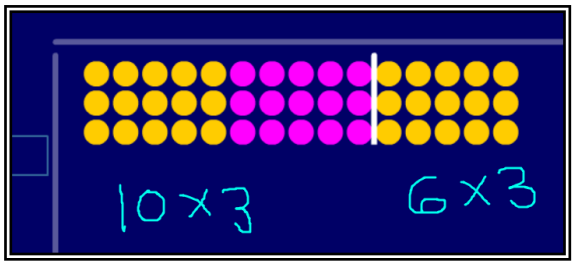
Also use **Multiarray ITP to model** how 16 is multiplied three times:
3 rows (groups) of 16 = 16 + 16 + 16

Also, use **Multiarray ITP to model** the 10 and the 6 both being multiplied by 3.

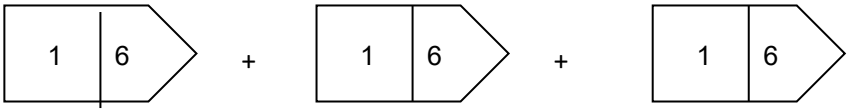


Year 3

so "10 grouped 3 times and 6 grouped 3 times"

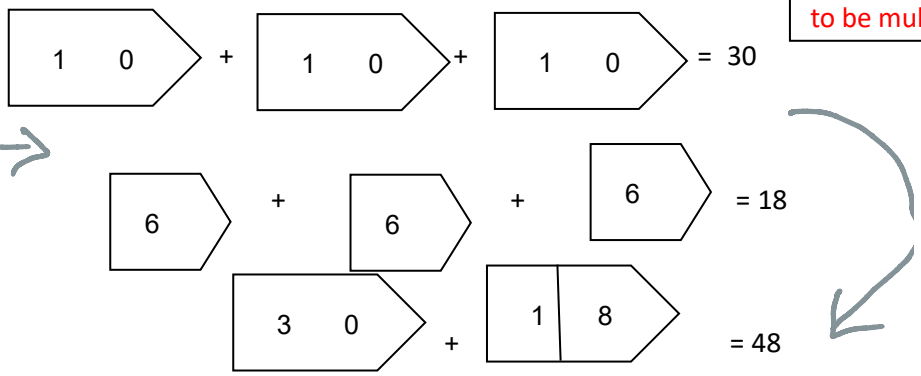


Partition 3 groups of 16:



So, partitioning out the 16 into 10 + 6 and multiplying both parts by 3.

Model and practise with **place value arrow cards and whiteboards** to show how each part of the number (partitioned part) needs to be multiplied by 3.



(using times-table facts or repeated addition)

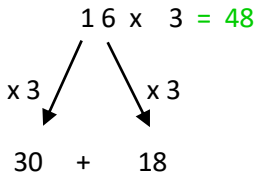
Year 3

Pupils do not need to record like this as it is inefficient (see next point for how to record).

Progression in Calculations

- Once partitioning concept understood, move onto recording as:

Year 3 onwards (mental strategy)



"10 multiplied by 3 is 30".

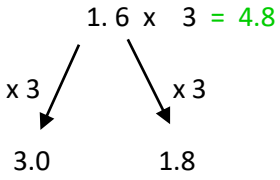
"6 multiplied by 3 is 18*

"30 + 18 = 48" (Only write this answer by the original calculation. Do not write it twice).

The arrows must point to the separate digits, the partitioned parts of the number: the tens and the ones.

This mental jotting can be used for all TU x U calculations.

This also works for decimals, e.g. 1.6 x 3



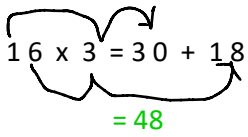
Years 5 & 6

0.6 x 3 is "6 tenths x 3 = 18 tenths
= 1.8" (one whole one and 8 tenths)

and/or using known facts that:
"0.6 x 3 is 10 times smaller than 6 x 3; 1.8 is 10 times smaller than 18."
This is NOT 'putting back' a decimal point!

If pupils do not have a clear understanding of tenths to be able to do this then then they are not ready for multiplication of decimals.
See addition of decimals for language/steps.

This becomes more efficient, in a similar way to adding two-digit numbers and can then be done without the arrows and, eventually, mentally, using known multiplication facts.



"10 x 3 is 30"
"6 x 3 is 18"

Year 3 onwards (mental strategy)

This mental jotting can be used for all TU x U calculations (from Year 3 and 4). However, pupils also need to know HOW to carry out short multiplication.

SHORT MULTIPLICATION is merely a formal way of writing down the above mental strategy (of partitioning to multiply) in a formal vertical presentation:

Expanded layout of short multiplication

1 6 x 3 = 48

```
  1 6
x   3
  1 8
  3 0
  4 8
```

"6 multiplied by 3 is 18* (or "3 multiplied by 6" now that the concept of commutativity is secure). Possibly, better to say "three sixes"

"10 multiplied by 3 is 30" (or "3 multiplied by 10"). Possibly, better to say "three tens"

"30 + 18 = 48"

Year 4: 2- or 3-digit x 1-digit numbers
Years 5 & 6: 4-digit x 1-digit numbers

Compact layout of short multiplication

1 6 x 3 = 48

```
  1 6
x   3
  4 8
  1
```

"3 sixes are 18"; place the 8 in the ones column, carry the ten.

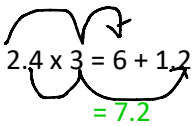
"3 tens are 30; add the carried ten is 40." Place the 4 (tens) in the tens column.

Remember: ALWAYS PRESENT CALCULATIONS HORIZONTALLY IN ORDER TO CONSIDER MENTAL CALCULATIONS FIRST.

Extend to decimals (Year 6):

2.4 x 3 = 7. 2 Either mentally (as above with arrow jottings as initial support) or by mentally partitioning to give:

Year 6 written method if not possible mentally/with jotting



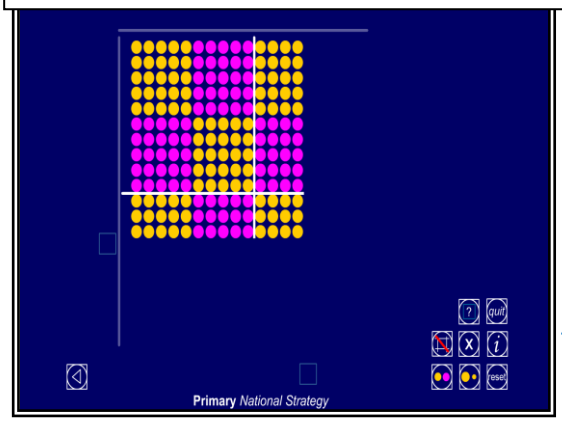
or 2.4
x 3
7.2
1

Put the decimal point in the equals bar in first.

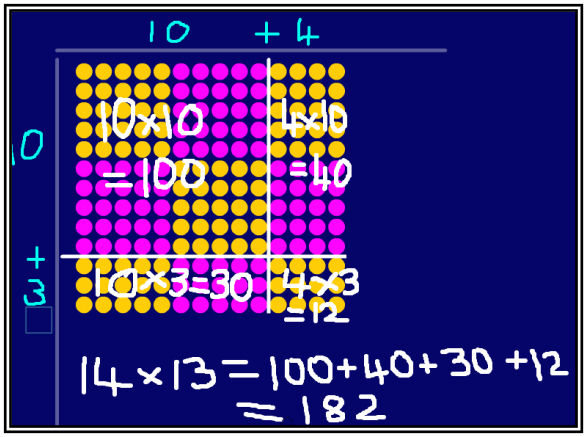
REMEMBER, short multiplication is a 'need-to-know' not a 'need-to-do' when other strategies (mental/jotting, e.g. arrows) are more efficient. Pupils must be encouraged to carry out the most appropriate, efficient AND reliable method for the numbers and for their abilities.

3. MULTIPLICATION GRID beginning with 2-digit x 2-digit numbers (Year 5+) **Year 5**

BEFORE moving onto written recording in a grid, it is essential to model the concept of partitioning to find the area of each rectangle in order to find the area of the whole. This visually represents what happens when we multiply two numbers i.e. find the number in an array – find the area of a rectangle.



Use Multiarray ITP to model partitioning into tens and ones, using the familiar visual pattern of 5s. Calculate the 'area' of each rectangle by multiplying the number of rows by the number of columns.



The link to **finding the area of a rectangle** is vital here so that the concept of multiplication is truly understood and not carried out by rote.

- Moving onto:

1 6 x 1 7 = 272

	1 0	+ 6
1 0	10 x 10 = 100	10 x 6 = 60
+ 7	7 x 10 = 70	7 x 6 = 42

100 + 70 + 60 + 42

170 + 102 = 272

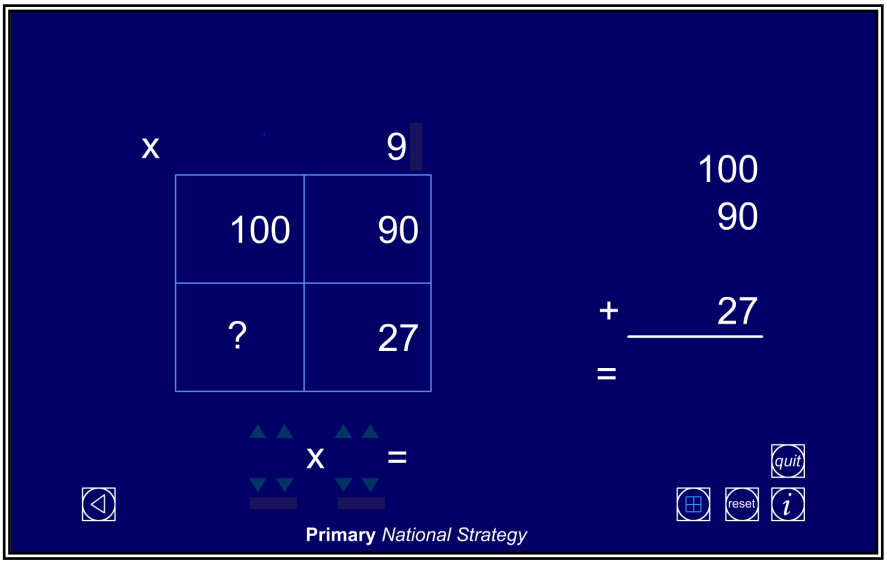
REMEMBER, ALWAYS PRESENT CALCULATIONS HORIZONTALLY IN ORDER TO CONSIDER MENTAL CALCULATIONS FIRST.

Again, if the calculation should be possible mentally then do not give it to practise vertical calculation, e.g. 23 x 15 should not be calculated vertically. Consider use of numbers carefully. Avoid numbers which involve x 2, x 4, x 5, x 8.

It is important to write the calculation in the grid for both the pupil and teacher to be able to identify errors made in multiplication facts or in the calculating the process.
It is also a reminder that the area of the rectangle is being calculated and the system is clear.

Try to adopt the system always starting each calculation with the number above (or to the side, as long as it is consistent) so that it is easy to check the correct calculations (and answers) have been carried out, e.g. ten and six x 10 and ten and

Where possible, use mental calculation strategies to calculate the total e.g. looking for known facts or adding the largest number first.



Year 5

Only use Multiplication grid ITP to assess understanding and application of the grid method by 'hiding' the question parts and 'revealing' some of the answer parts.

PROGRESSION:

1. TU x TU
2. HTU x U
3. HTU x TU
4. HTU x HTU
5. U.t x U

Revert back to this stage (the grid method) when using decimals, even if at a higher stage of progression for multiplication of whole numbers, as it helps clarify the size of decimals and gives a clear visual image to emphasise the place value involved.

Again, if the calculation should be possible mentally then do not give it to practise vertical calculation, e.g. TU x 15 should not be calculated vertically (see p.10 for mental jottings). Consider use of numbers carefully, including the differentiation steps within a stage. Avoid numbers which involve x 2, x 4, x 5 or x 8. When secure, teach pupils to choose the most appropriate method and include calculations that can be carried out mentally/with a jotting so that pupils 'spot' them.

Avoid incorrect use of mathematical language, e.g. correct the commonly-used "timesing" to "multiplying".

4. EXPANDED LONG MULTIPLICATION

1 6 X 1 7 = 272

```
  1 6
x 1 7
  1 0 0   10 X 10
  6 0     10 X  6
  7 0      7 X 10
  4 2      7 X  6
-------
  2 7 2
```

Initially, teach alongside the multiplication grid and show how each calculation links to each 'box' of the multiplication grid.

Tip: If TU x TU (i.e. 2-digit x 2-digit) then a '2 x 2' multiplication needs 4 calculations/ boxes (2 x 2 = 4). If HTU x TU (3-digit x 2-digit) then a '3 x 2' multiplication needs 6 calculations (3 x 2 = 6) etc.

Check that the correct calculations have been made by looking for the numbers involved (on the right) i.e. 10 x 16 (the first two) and 7 x 16 (the second two).

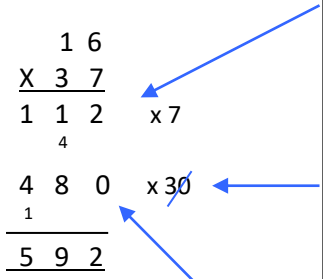
Where possible, use mental calculation strategies to calculate the total e.g. looking for known facts or adding the largest number first.

Year 5

Progression in Calculations

- Moving onto the formal compact method of **LONG MULTIPLICATION** **Years 5 & 6**

1 6 x 3 7 = 272



Year 5: 3- then 4-digit x 2-digit numbers

Year 6: 4-digit x 2-digit numbers

"7 times 6 is 42; carry the 4 tens (put 4 below in the tens column) and put the 2 in the ones column.
7 times 1 ten is 7 tens (70); plus the 4 carried tens is 11 tens so 110 so 1 in the hundreds column and 1 in the tens column."

Then, as we are multiplying by multiples of tens next, we know there are no ones so already know there will be a zero in the ones column on the next row. After we have multiplied by 10 (placed the zero in the ones column to indicate no ones for a multiple of ten), cross out the zero in x 30 to remind that it has been done.
We do NOT just "put a zero." It is important that it is understood why there are no ones. We have multiplied by ten.

Having multiplied by 10, we can now multiply by (in this example) 3.

3 x 6 = 18 so carry the ten (ten tens, 100) in the hundreds column and put the 8 tens in the tens column. Then 3 x 10 is 30 (30 tens, 300) plus the carried one (hundred) is 4 (hundred)/

Add up the columns from right to left in line with column addition procedures.

Again, where possible, use mental calculation strategies to multiply, e.g. if x 17 then just multiply the whole number by 10 not each part for the second row calculation; for x 26, multiply by 20 by doubling and multiplying by 10 in one go rather than multiplying each partitioned part by 2 and 'moving up a place so there is a zero in the ones column).

Tip: The number with the fewest digits goes below the other number. The number of digits determines the number of rows of calculations, e.g. 35 x 452 so 35 goes below the 452 and two rows of calculations will be needed – one calculation for x 5 and the other for x 30.

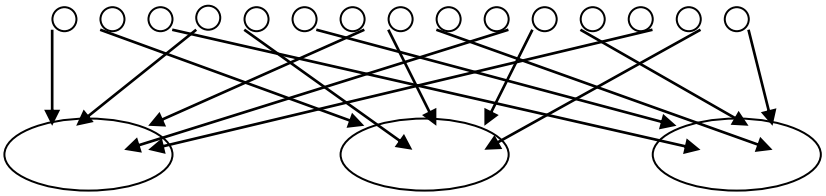
NB: Children need to write the carried digits in small writing on the line, NOT use a whole row for the carried digits as it appears above.

NB: IT IS NEVER NECESSARY TO DO A ROW TO MULTIPLY BY ZERO!

DIVISION

1. SHARING

15 ÷ 3



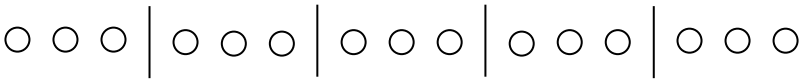
"15 shared between 3"

5 + 5 + 5 or 3 sets of 5
or 5 'each'

Year 2

2. GROUPING

15 ÷ 3

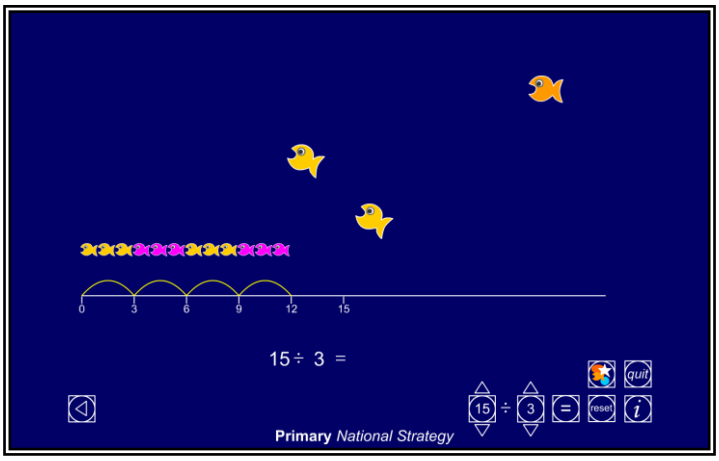


"15 grouped in 3s"

3 + 3 + 3 + 3 + 3 or 5 groups of 3

Grouping can be easily modelled on the 100-bead bar.

ONCE LARGER NUMBERS ARE USED, GROUPING IS MORE EFFICIENT AND RELIABLE THAN SHARING.

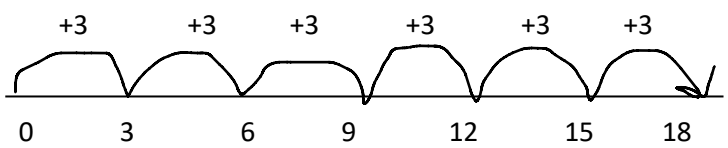


Repeated addition (easier to count on than count back).

Model using the Grouping ITP to make the link between grouping on the number line and repeated addition.

Grouping on a number line should NEVER be used for known times-tables facts as it would be a pointless exercise if the related division fact is known mentally. Nor should it be used if the child can count on in steps of the divisor, e.g. 3, 6, 9, as this will be quicker on their fingers than constructing a number line. Nor should it be used for dividing by 2, 4, 5 or 8 as there are mental strategies for these. Consider numbers and efficiency of calculation. Encourage decision-making so that children do not feel they HAVE to use a number line when they count up in the heads more quickly.

18 ÷ 3 = 6

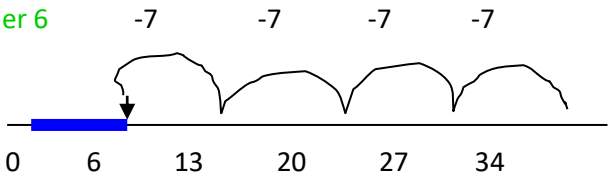


showing '6 groups of 3'

Progression in Calculations

- Moving onto remainders:

34 ÷ 7 = 4 remainder 6



shown as '4 groups of 7 and 6 left over'

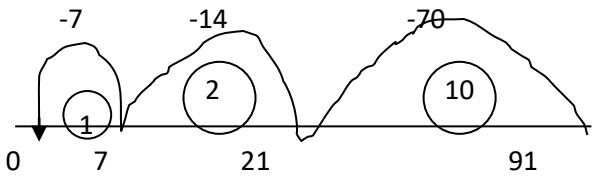
Repeated subtraction now shows the remainder more clearly than repeated addition as the remainder/left over is less than the divisor and no calculation is required to see how much is left over. It also helps avoid pupils making another group whatever the size of the reminder.

Year 3 for unknown times tables

- Moving onto division calculations involving larger numbers by **'CHUNKING' ON A NUMBER LINE** and using a **'Help Box' of key facts** based on best friend (10) and second-best friend (h/d).

Concrete representation: Multiples of the divisor are subtracted along the number line. This can be modelled by relating it to breaking of chunks (rows) of a chocolate bar, rather than cube-by-cube.

91 ÷ 7 = 13 (10 groups + 2 groups + 1 group)



Help Box

10	70
5	35
2	14
1	7

The number of 'chunks' of the divisor being subtracted is written in a circle below the jump. The total of these numbers is the number of groups of the divisor in the larger number i.e. the answer.

- The Help Box ONLY needs x 10, x 5, x2 and x1 facts of the divisor. All other facts can be derived from these.
- Remember, x5 is calculated mentally by halving x10 and x2 is expressed as 'double'.
- By drawing the Help Box in a cloud (after the facts have been listed) it stays tidy by not being crooked lines of a box and does not slow down the process.
- The Help Box MUST be used before the calculation is carried out. If it is not needed then the calculation is probably too easy and the pupil could probably calculate by mentally chunking the number.

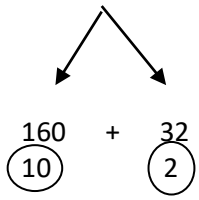
PROGRESSION:

1. TU ÷ U where the dividend is greater than the 12th multiple of the divisor so that it is not within known tables facts
2. HTU ÷ U

Progression in Calculations

- **MENTAL CHUNKING**

192 ÷ 16 = 12



Year 4 onwards

Partitioning 192 into the 10th multiple of the divisor and "some more" and recording the number of groups of the divisor in circles below.

- **MENTAL CHUNKING** leads to **SHORT DIVISION** when it is understood that it is multiples of ten (or hundred etc.) of the divisor and not single digits multiplied by the divisor.

657 ÷ 3 = 239

$$\begin{array}{r} 219 \\ 3\overline{)657} \end{array}$$

512 ÷ 16 = 32

$$\begin{array}{r} 032 \\ 16\overline{)512} \end{array}$$

1. "16 does not go into 500 a hundred times." Put a small 0 in the hundreds column to ensure the place is not used.
2. "16 goes into 51 (tens) three (tens) times, which is 48 (tens)" .. knowing three 16s are 48. Put a 3 in the tens column above the line to show this.
3. 51 tens – 48 tens = 3 tens remaining – show as a small, higher 3 in the tens column below the line.
4. "16 goes into 32 twice." Put a 2 in the ones column above the line.

Years 5 & 6: 3- then 4-digit ÷ 1-digit

- Moving onto **'VERTICAL CHUNKING'** for larger numbers e.g. HTU ÷ TU

192 ÷ 24 = 8

192	
-120	5
72	
-48	2
24	
-24	1
0	

10	240
5	120
2	48
1	24

Year 6:
4-digit ÷ 2-digit numbers

Subtraction calculations involved when removing each chunk of the divisor SHOULD NOT be carried out vertically – this is when most children make errors with division. Instead, calculate mentally (by counting up) or by using a jotting to count up on a number line.

The Help Box must still be used with the same key facts, based around 10, 5, 2 and 1. Other related facts may be calculated mentally from these e.g. x20 is double x10 but need not be written in the Help Box. When times-table/division facts are spotted (once the method is clearly understood and used with the appropriate use of mathematical language in explaining the process) then these maybe used to make the process more efficient.

The end of the SHORT OR LONG DIVISION process is when the calculation ends in zero or a number less than the divisor. If there is a remainder, this should be expressed as a fraction,
e.g. 292 ÷ 13 = 22 remainder 6
= 22 6/13

If the fraction's decimal equivalent is known then the decimal is used,

e.g. 203 ÷ 14 = 14 7/14
= 14.5

TIP: WHEN TO USE SHORT DIVISION AND WHEN TO USE LONG DIVISION – DECISION MAKING:

If the 2-digit divisor is less than the first two digits of the dividend then use short division.

If the divisor is greater than the first two digits of the dividend then use long multiplication.

THE TERM 'BUS STOP METHOD' SHOULD NOT BE USED! It is vital that children know the difference between *short division* and *long division* and when to choose which method, depending on the size of the numbers. A generic 'nickname' of 'bus stop method' does not indicate which type of division is being used.

Progression in Calculations

- VERTICAL CHUNKING leads to **LONG DIVISION**

Long division relies on the ability to multiply 2-digit numbers (when the divisor is a 2-digit number) which can be difficult. Using best friend (10) and second-best friend (h/d) facts helps to estimate and get closer to the answer without endless 'trial and improvement' long multiplication attempts to work out how many times the divisor 'goes into' the number.

Each time, aiming to subtract the largest possible multiple of the divisor.

Year 6:
4-digit ÷ 2-digit numbers

1440 ÷ 32 = 45

```
  0045
32|1440
 -128
  160
 -160
    0
```

At the side of the page, write the number of possible groups (in circles):
Always calculate 10x, 5x, 2x and 1x the divisor before starting.

10	320	
5	160	(using ½ of 32 x 10)
2	64	(double the divisor)
1	32	

If these numbers are not large/small enough to subtract then use d/h to calculate other multiples of the divisor and using what has already been calculated, e.g. double 2x is 4x:

4 128 (using double 64)

If 8x was needed, we could use double 4x. If 6x was needed, we could use 5x plus 1x the divisor and so on.

If the numbers at the top are not put there until the end, the workings/estimations at the side mean that the multiples are not forgotten/lost.

PROGRESSION:

1. HTU ÷ TU
2. ThHTU ÷ TU

THE TERM 'BUS STOP METHOD' SHOULD NOT BE USED! It is vital that children know the difference between *short division* and *long division* and when to choose which method, depending on the size of the numbers. A generic 'nickname' of 'bus stop method' does not indicate which type of division is being used.

REMINDER TIP: WHEN TO USE SHORT DIVISION AND WHEN TO USE LONG DIVISION – DECISION MAKING:

If the 2-digit divisor is less than the first two digits of the dividend then use short division.

If the divisor is greater than the first two digits of the dividend then use long multiplication.

Bronze, Silver and Gold Approach to Learning Addition and Subtraction Facts

<table>
<tr><th>Year</th><th>Facts to be Learnt – Instant Recall
(including knowing and using the commutative rule for + and related facts for -)</th><th>BSG Stages</th></tr>
<tr><td>Rec.</td><td>
Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5, including subtraction facts) and some number bonds to 10 (see Year 1).
<table>
<tr><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th></tr>
<tr><td>0 + 1</td><td>0 + 2
1 + 1</td><td>0 + 3
1 + 2</td><td>0 + 4
1 + 3
2 + 2</td><td>0 + 5
1 + 4
2 + 3</td></tr>
<tr><td>1 - 0</td><td>2 – 0
2 – 1
2 - 2</td><td>3 – 0
3 – 1
3 – 2
3 - 3</td><td>4 – 0
4 – 1
4 – 2
4 – 3
4 - 4</td><td>5 – 0
5 – 4
5 – 3
5 – 2
5 - 1
5 - 5</td></tr>
</table>
<table>
<tr><th colspan="4">10</th></tr>
<tr><td>0 + 10
1 + 9
2 + 8</td><td>3 + 7
4 + 6
5 + 5</td><td>10 – 0
10 – 1
10 – 2
10 – 3
10 - 4</td><td>10 – 5
10 – 6
10 – 7
10 – 8
10 – 9
10 - 10</td></tr>
</table>
</td><td>
Bronze: addition facts,
e.g. 3 + 2 = 5 so 2 + 3 = 5

Silver: related subtraction facts,
e.g. 4 – 1 = 3 and 4 – 3 = 1

Gold: empty boxes,
e.g. 5 = □ + 2 2 + □ = 5
</td></tr>
</table>

Bronze, Silver and Gold Approach to Learning Addition and Subtraction Facts

<table>
<tr><th>Year</th><th>Facts to be Learnt – Instant Recall
(including knowing and using the commutative rule for + and related facts for -)</th><th>BSG Stages</th></tr>
<tr><td rowspan="4">Year 1</td><td colspan="2">Consolidate all previous objectives.</td></tr>
<tr><td>Derive and recall all pairs of numbers that <u>total</u> 10 (‘have a sum of 10’) and understand the commutative relationship, e.g. 1 + 9 = 9 + 1
<table><tr><th colspan="4">10</th></tr><tr><td>0 + 10
1 + 9
2 + 8</td><td>3 + 7
4 + 6
5 + 5</td><td>10 – 0
10 – 1
10 – 2
10 – 3
10 - 4</td><td>10 – 5
10 – 6
10 – 7
10 – 8
10 – 9
10 - 10</td></tr></table></td><td>Bronze: addition facts,
e.g. 3 + 7 = 10 so 7 + 3 = 10
Silver: related subtraction facts,
e.g. 10 – 4 = 6 and 10 – 6 = 4
Gold: empty boxes,
e.g. 10 = □ + 7 2 + □ = 10
□ = 5 + 5</td></tr>
<tr><td>Derive and recall all facts <u>within</u> 10 and understand the commutative relationship, e.g. 2 + 6 = 6 + 2
<table><tr><th>6</th><th>7</th><th>8</th><th>9</th></tr><tr><td>0 + 6
1 + 5
2 + 4
3 + 3</td><td>0 + 7
1 + 6
2 + 5
3 + 4</td><td>0 + 8
1 + 8
2 + 6
3 + 5
4 + 4</td><td>0 + 9
1 + 8
2 + 7
3 + 6
4 + 5</td></tr><tr><td>6 – 0
6 – 1
6 – 2
6 – 3
6 – 4
6 – 5
6 - 6</td><td>7 – 0
7 – 1
7 – 2
7 – 3
7 – 4
7 – 5
7 – 6
7 - 7</td><td>8 – 0
8 – 1
8 – 2
8 – 3
8 – 4
8 – 5
8 – 6
8 – 7
8 - 8</td><td>9 – 0
9 – 1
9 – 2
9 – 3
9 – 4
9 – 5
9 – 6
9 – 7
9 – 8
9 - 9</td></tr></table></td><td>Bronze: addition facts,
e.g. 2 + 7 = 9 so 7 + 2 = 9
Silver: related subtraction facts,
e.g. 7 – 3 = 4 and 7 – 4 = 3
Gold: empty boxes,
e.g. 8 = □ + 2 6 - □ = 4 □ = 4 + 3</td></tr>
<tr><td>Add and subtract a multiple of 10 from a two-digit number
e.g. 23 + 10 = 33 63 – 10 = 53</td><td>Bronze: addition facts,
e.g. 35 + 10 = 45 so 10 + 35 = 45
Silver: related subtraction facts,
e.g. 37 – 10 = 27
Gold: empty boxes,
e.g. 10 + □ = 32 34 - □ = 24
54 = □ - 10</td></tr>
</table>

Bronze, Silver and Gold Approach to Learning Addition and Subtraction Facts

<table>
<tr><th>Year</th><th>Facts to be Learnt – Instant Recall
(including knowing and using the commutative rule for + and related facts for -)</th><th>BSG Stages</th></tr>
<tr><td rowspan="3">Year 2</td><td colspan="2">Consolidate all previous objectives.</td></tr>
<tr><td>Derive and recall all pairs of multiples of 10 with totals up to 100 and understand the commutative relationship, e.g. 30 + 70 = 70 + 30

<table>
<tr><th colspan="4">100</th></tr>
<tr><td>0 + 100
10 + 90
20 + 80</td><td>30 + 70
40 + 60
50 + 50</td><td>100 - 0
100 – 10
100 – 20
100 – 30
100 - 40</td><td>100 – 50
100 – 60
100 – 70
100 – 80
100 – 90
100 - 100</td></tr>
</table></td><td>Bronze: addition facts,
e.g. 30 + 70 = 100 so 70 + 30 = 100

Silver: related subtraction facts,
e.g. 100 – 80 = 20

Gold: empty boxes,
e.g. 100 - □ = 10 100 = □ + 40 □ = 100 - 30</td></tr>
<tr><td>Derive and recall all pairs with totals to 20 (‘have a sum of 20’) and understand the commutative relationship, e.g. 5 + 15 = 15 + 5

<table>
<tr><th colspan="4">20</th></tr>
<tr><td>0 + 20
1 + 19
2 + 18
3 + 17
4 + 16
5 + 15
6 + 14
7 + 13
8 + 12
9 + 11
10 + 10</td><td>11 + 9
12 + 8
13 + 7
14 + 6
15 + 5
16 + 4
17 + 3
18 + 2
19 + 1
20 + 0</td><td>20 – 0
20 – 1
20 – 2
20 – 3
20 – 4
20 – 5
20 – 6
20 – 7
20 – 8
20 – 9
20 - 10</td><td>20 – 11
20 – 12
20 – 13
20 – 14
20 – 15
20 – 16
20 – 17
20 – 18
20 – 19
20 – 20</td></tr>
</table></td><td>Bronze: addition facts,
e.g. 17 + 3 = 20 so 3 + 17 = 20

Silver: related subtraction facts,
e.g. 20 – 14 = 6

Gold: empty boxes,
e.g. 20 - □ = 16 20 = □ + 11 □ = 20 - 7</td></tr>
</table>

Bronze, Silver and Gold Approach to Learning Addition and Subtraction Facts

<table>
<tr><th>Year</th><th>Facts to be Learnt – Instant Recall
(including knowing and using the commutative rule for + and related facts for -)</th><th>BSG Stages</th></tr>
<tr><td rowspan="2">Year 3</td><td colspan="2">Consolidate all previous objectives.</td></tr>
<tr><td>Derive and recall all addition and subtraction facts for each number within 15 and understand the commutative relationship, e.g. 4 + 11 = 11 + 4
<table>
<tr><th>11</th><th>12</th><th>13</th><th>14</th><th>15</th></tr>
<tr><td>0 + 1
1 + 10
2 + 9
3 + 8
4 + 7
5 + 6</td><td>0 + 12
1 + 11
2 + 10
3 + 9
4 + 8
5 + 7
6 + 9</td><td>0 + 13
1 + 12
2 + 11
3 + 10
4 + 9
5 + 9
6 + 7</td><td>0 + 14
1 + 13
2 + 12
3 + 11
4 + 10
5 + 9
6 + 8
7 + 7</td><td>0 + 15
1 + 14
2 + 13
3 + 12
4 + 11
5 + 10
6 + 9
7 + 8</td></tr>
<tr><td>11 – 0
11 – 1
11 – 2
11 – 3
11 – 4
11 – 5
11 – 6
11 – 7
11 – 8
11 = 9
11 – 10
11 - 11</td><td>12 – 0
12 – 1
12 – 2
12 – 3
12 – 4
12 – 5
12 – 6
12 – 7
12 – 8
12 – 9
12 – 10
12 – 11
12 – 12</td><td>13 – 0
13 – 1
13 – 2
13 – 3
13 –4
13 – 5
13 – 6
13 – 7
13 – 8
13 – 9
13 – 10
13 – 11
13 – 12
13 – 13</td><td>14 – 0
14 – 1
14 – 2
14 – 3
14 – 4
14 – 5
14 – 6
14 – 7
14 – 8
14 – 9
14 – 10
14 – 11
14 – 12
14 – 13
14 – 14</td><td>15 – 0
15 – 1
15 – 2
15 – 3
15 – 4
15 – 5
15 – 6
15 – 7
15 – 8
15 – 9
15 – 10
15 – 11
15 – 12
15 – 13
15 – 14
15 - 15</td></tr>
</table></td><td>Bronze: addition facts,
e.g. 9 + 3 = 12 so 3 + 9 = 12
Silver: related subtraction facts,
e.g. 14 – 8 = 6
Gold: empty boxes,
e.g. 13 - □ = 7 15 = □ + 9
□ = 13 - 7</td></tr>
</table>

Bronze, Silver and Gold Approach to Learning Addition and Subtraction Facts

<table>
<tr><th>Year</th><th>Facts to be Learnt – Instant Recall
(including knowing and using the commutative rule for + and related facts for -)</th><th>BSG Stages</th></tr>
<tr><td>Year 3 contd.</td><td>
Derive and recall all addition and subtraction facts for each number within 20 and understand the commutative relationship, e.g. 3 + 14 = 14 + 3
<table>
<tr><th>16</th><th>17</th><th>18</th><th>19</th></tr>
<tr><td>0 + 16
1 + 15
2 + 14
3 + 13
4 + 12
5 +11
6 + 10
7 + 9
8 + 8</td><td>0 + 12
1 + 11
2 + 10
3 + 9
4 + 8
5 + 7
6 + 9</td><td>0 + 13
1 + 12
2 + 11
3 + 10
4 + 9
5 + 9
6 + 7</td><td>0 + 14
1 + 13
2 + 12
3 + 11
4 + 10
5 + 9
6 + 8
7 + 7</td></tr>
<tr><td>11 – 0
11 – 1
11 – 2
11 – 3
11 – 4
11 – 5
11 – 6
11 – 7
11 – 8
11 = 9
11 – 10
11 - 11</td><td>12 – 0
12 – 1
12 – 2
12 – 3
12 – 4
12 – 5
12 – 6
12 – 7
12 – 8
12 – 9
12 – 10
12 – 11
12 – 12</td><td>13 – 0
13 – 1
13 – 2
13 – 3
13 –4
13 – 5
13 – 6
13 – 7
13 – 8
13 – 9
13 – 10
13 – 11
13 – 12
13 – 13</td><td>14 – 0
14 – 1
14 – 2
14 – 3
14 – 4
14 – 5
14 – 6
14 – 7
14 – 8
14 – 9
14 – 10
14 – 11
14 – 12
14 – 13
14 – 14</td></tr>
</table>
</td><td>
Bronze: addition facts,
e.g. 12 + 5 = 17 so 5 + 12 = 17

Silver: related subtraction facts,
e.g. 18 – 14 = 4

Gold: empty boxes,
e.g. 19 - □ = 8 16 = □ + 11
</td></tr>
</table>

Bronze, Silver and Gold Approach to Learning Addition and Subtraction Facts

<table>
<tr><th>Year</th><th>Facts to be Learnt – Instant Recall
(including knowing and using the commutative rule for + and related facts for -)</th><th>BSG Stages</th></tr>
<tr><td rowspan="2">Year 3 contd.</td><td>Derive and recall sums (+) and differences (-) of multiples of 10 and understand the commutative relationship, e.g. 30 + 40 = 40 + 30
<table>
<tr><th>20</th><th>30</th><th>40</th><th>50</th></tr>
<tr><td>0 + 20
10 + 10
20 + 0</td><td>0 + 30
10 + 20</td><td>0 + 40
10 + 30
20 + 20</td><td>0 + 50
10 + 40
20 + 30</td></tr>
<tr><td>20 – 0
20 – 10
20 - 20</td><td>30 – 0
30 – 10
30 – 20
30 - 30</td><td>40 – 0
40 – 10
40 – 20
40 – 30
40 - 40</td><td>50 – 0
50 – 10
50 – 20
50 – 30
50 – 40
50 - 50</td></tr>
<tr><th>60</th><th>70</th><th>80</th><th>90</th></tr>
<tr><td>0 + 60
10 + 50
20 + 40
30 + 30</td><td>0 + 70
10 + 60
20 + 50
30 + 40</td><td>0 + 80
10 + 70
20 + 60
30 + 50
40 + 40</td><td>0 + 90
10 + 80
20 + 70
30 + 60
40 + 50</td></tr>
<tr><td>60 – 0
60 - 10
60 – 20
60 – 30
60 – 40
60 – 50
60 - 60</td><td>70 - 0
70 – 10
70 – 20
70 – 30
70 – 40
70 – 50
70 – 60
70 - 70</td><td>80 – 0
80 – 10
80 – 20
80 – 30
80 – 40
80 – 50
80 – 60
80 – 70
80 - 80</td><td>90 – 0
90 – 10
90 – 20
90 – 30
90 – 40
90 – 50
90 – 60
90 – 70
90 – 80
90 - 90</td></tr>
</table></td><td>Bronze: addition facts,
e.g. 20 + 30 = 50 so 30 + 20 = 50
Silver: related subtraction facts,
e.g. 70 – 40 = 30
Gold: empty boxes,
e.g. 80 - □ = 60 90 = □ + 30
□ = 50 - 30</td></tr>
<tr><td>Derive and recall all pairs that total 100, e.g. 32 + 68
(using the knowledge that the tens need to total 90 and the ones need to total 10).</td><td>Bronze: addition facts,
e.g. 37 + 63 = 100 so 63 + 37 = 100
Silver: related subtraction facts,
e.g. 100 – 14 = 86
Gold: empty boxes,
e.g. 100 - □ = 76 100 = □ + 21
□ = 62 + 38</td></tr>
</table>

Bronze, Silver and Gold Approach to Learning Addition and Subtraction Facts

<table>
<tr><th>Year</th><th>Facts to be Learnt – Instant Recall
(including knowing and using the commutative rule for + and related facts for -)</th><th>BSG Stages</th></tr>
<tr><td rowspan="3">Year 4</td><td colspan="2">Consolidate all previous objectives.</td></tr>
<tr><td>Derive sums and differences of pairs of multiples of 100 or 1000
<table>
<tr><th>200</th><th>300</th><th>400</th><th>500</th></tr>
<tr><td>0 + 200
100 + 100
200 + 00</td><td>0 + 300
100 + 200</td><td>0 + 400
100 + 300
200 + 200</td><td>0 + 500
100 + 400
200 + 300</td></tr>
<tr><td>200 – 0
200 – 100
200 - 200</td><td>300 – 0
300 – 100
300 – 200
300 - 300</td><td>400 – 0
400 – 100
400 – 200
400 – 300
400 - 400</td><td>500 – 0
500 – 100
500 – 200
500 – 300
500 – 400
500 - 500</td></tr>
<tr><th>600</th><th>700</th><th>800</th><th>900</th></tr>
<tr><td>0 + 600
100 + 500
200 + 400
300 + 300</td><td>0 + 70
100 + 600
200 + 500
300 + 400</td><td>0 + 80
100 + 700
200 + 600
300 + 500
400 + 400</td><td>0 + 900
100 + 800
200 + 700
300 + 600
400 + 500</td></tr>
<tr><td>600 – 0
600 - 100
600 – 200
600 – 300
600 – 400
600 – 500
600 - 600</td><td>700 - 0
700 – 100
700 – 200
700 – 300
700 – 400
700 – 500
700 – 600
700 - 700</td><td>800 – 0
800 – 100
800 – 200
800 – 300
800 – 400
800 – 500
800 – 600
800 – 700
800 - 800</td><td>900 – 0
900 – 100
900 – 200
900 – 300
900 – 400
900 – 500
900 – 600
900 – 700
900 – 800
900 - 900</td></tr>
</table></td><td>Bronze: addition facts,
e.g. 200 + 300 = 500 so 300 + 200 = 500; 2000 + 3000 = 5000 so 3000 + 2000 = 5000

Silver: related subtraction facts,
e.g. 700 – 400 = 300; 7000 – 4000 = 3000

Gold: empty boxes,
e.g. 800 - □ = 600 900 = □ + 300
8000 - □ = 6000 9000 = □ + 3000</td></tr>
<tr><td colspan="2">Add or subtract mentally pairs of two-digit whole numbers,
e.g. 47 + 58, 91 - 35</td><td>Bronze: addition facts,
e.g. 57 + 26 = 83 so 26 + 57 = 83

Silver: related subtraction facts,
e.g. 64 – 28 = 36

Gold: empty boxes,
e.g. 69 - □ = 41 56 = □ + 17</td></tr>
</table>

Bronze, Silver and Gold Approach to Learning Addition and Subtraction Facts

Year	Facts to be Learnt – Instant Recall (including knowing and using the commutative rule for + and related facts for -)	BSG Stages
Year 5	**Consolidate all previous objectives.**	
	Derive sums and differences of decimals, e.g. 6.5 + 2.7, 3.9 - 1.7	**Bronze**: addition facts, e.g. 2.7 + 3.9 = 6.6 so 3.9 + 2.7 = 6.6 **Silver**: related subtraction facts, e.g. 7.8 – 4.3 = 3.5 **Gold**: empty boxes, e.g. 8.5 - □ = 6.9 9.3 = □ + 3.2
Year 6	**Consolidate all previous objectives.**	**Platinum: Apply KS2 +/- facts/objectives within problem solving contexts, e.g. measure;** use these instantly known facts instead of inefficient vertical written methods.

Bronze, Silver and Gold Approach to Learning Multiplication and Division Facts

<table>
<tr><th>Year</th><th>Facts to be Learnt – Instant Recall
(including knowing and using the commutative rule for x and related facts for ÷)</th><th>BSG Stages</th></tr>
<tr><td>Rec.</td><td>
Automatically recall (without reference to rhymes, counting or other aids) some number bonds to 10, including double facts.

Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.

<table>
<tr><th colspan="6">Doubling Facts to 10</th></tr>
<tr><td>0 + 0 = 0</td><td>1 + 1 = 2</td><td>2 + 2 = 4</td><td>3 + 3 = 6</td><td>4 + 4 = 8</td><td>5 + 5 = 10</td></tr>
<tr><td>double 0 is 0</td><td>double 1 is 2</td><td>double 2 is 4</td><td>double 3 is 6</td><td>double 4 is 8</td><td>double 5 is 10</td></tr>
</table>
<table>
<tr><th colspan="5">Extend to Halving Facts Within 10</th></tr>
<tr><td>half of 2 is 1</td><td>half of 4 is 2</td><td>half of 6 is 3</td><td>half of 8 is 4</td><td>half of 10 is 5</td></tr>
</table>
</td><td>
Bronze: doubling facts,
e.g. 2 + 2 = 4 so double 2 is 4

Silver: halving facts,
e.g. half of 6 is 3
half of 10 = 5

Gold: related facts,
e.g. double ☐ = 8
☐ = half of 8

half of ☐ = 5
☐ = half of 10
</td></tr>
<tr><td colspan="3"></td></tr>
<tr><td rowspan="5">Year 1</td><td colspan="2">Know and consolidate all previous objectives.</td></tr>
<tr><td>Count in 10s from zero (to the 12th multiple).
0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 120, 120.</td><td>Bronze for Year 2, ten times table.</td></tr>
<tr><td>Count in 2s from zero (to the 12th multiple).
0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24.</td><td>Bronze for Year 2, two times table.</td></tr>
<tr><td>Count in 5s from zero (to the 12th multiple).
0, 5, 10, 15, 20, 25, 30, 35, 40, 50, 55, 60.</td><td>Bronze for Year 2, five times table.</td></tr>
<tr><td>
Recall doubles up to 10 + 10.

<table>
<tr><th colspan="6">Doubling Facts to 20</th></tr>
<tr><td>0 + 0 = 0</td><td>1 + 1 = 2</td><td>2 + 2 = 4</td><td>3 + 3 = 6</td><td>4 + 4 = 8</td><td>5 + 5 = 10</td></tr>
<tr><td>double 0 is 0</td><td>double 1 is 2</td><td>double 2 is 4</td><td>double 3 is 6</td><td>double 4 is 8</td><td>double 5 is 10</td></tr>
<tr><td>6 + 6 = 12</td><td>7 + 7 = 14</td><td>8 + 8 = 16</td><td>9 + 9 = 18</td><td>10 + 10 = 20</td><td></td></tr>
<tr><td>double 6 is 12</td><td>double 7 is 14</td><td>double 8 is 16</td><td>double 9 is 18</td><td>double 10 is 20</td><td></td></tr>
</table>
Recall halves of even numbers up to half of 20.

<table>
<tr><th colspan="5">Halving Facts Within 20</th></tr>
<tr><td>half of 2 is 1</td><td>half of 4 is 2</td><td>half of 6 is 3</td><td>half of 8 is 4</td><td>half of 10 is 5</td></tr>
<tr><td>half of 12 is 6</td><td>half of 14 is 7</td><td>half of 16 is 8</td><td>half of 18 is 9</td><td>half of 20 is 10</td></tr>
</table>
</td><td>
Bronze: doubling facts,
e.g. 8 + 8 = 16 so double 8 is 16

Silver: halving facts,
e.g. half of 14 is 7
half of 20 = 10

Gold: related facts,
e.g. double ☐ = 18
☐ = half of 18

half of ☐ = 6
☐ = half of 12
</td></tr>
<tr><td colspan="3"></td></tr>
</table>

Bronze, Silver and Gold Approach to Learning Multiplication and Division Facts

<table>
<tr><th>Year</th><th>Facts to be Learnt – Instant Recall
(including knowing and using the commutative rule for x and related facts for ÷)</th><th>BSG Stages</th></tr>
<tr><td rowspan="4">Year 2</td><td colspan="2">Know and consolidate all previous objectives.</td></tr>
<tr><td>Count in 3s from zero (to the 12th multiple).
0, 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36.</td><td>Bronze for Year 3, three times table.</td></tr>
<tr><td>Recall multiplication and division facts for the 10 times table.
<table><tr><th colspan="3">The TEN Times Table</th></tr><tr><th>Multiplication Facts</th><th></th><th>Division Facts</th></tr><tr><td>10 x 0 = 0</td><td>0</td><td>0 ÷ 10 = 0</td></tr><tr><td>10 x 1 = 10</td><td>1</td><td>10 ÷ 10 = 1</td></tr><tr><td>10 x 2 = 20</td><td>2</td><td>20 ÷ 10 = 2</td></tr><tr><td>10 x 3 = 30</td><td>3</td><td>30 ÷ 10 = 3</td></tr><tr><td>10 x 4 = 40</td><td>4</td><td>40 ÷ 10 = 4</td></tr><tr><td>10 x 5 = 50</td><td>5</td><td>50 ÷ 10 = 5</td></tr><tr><td>10 x 6 = 60</td><td>6</td><td>60 ÷ 10 = 6</td></tr><tr><td>10 x 7 = 70</td><td>7</td><td>70 ÷ 10 = 7</td></tr><tr><td>10 x 8 = 80</td><td>8</td><td>80 ÷ 10 = 8</td></tr><tr><td>10 x 9 = 90</td><td>9</td><td>90 ÷ 10 = 9</td></tr><tr><td>10 x 10 = 100</td><td>10</td><td>100 ÷ 10 = 10</td></tr><tr><td>10 x 11 = 110</td><td>11</td><td>110 ÷ 10 = 11</td></tr><tr><td>10 x 12 = 120</td><td>12</td><td>120 ÷ 10 = 12</td></tr></table></td><td>Bronze – I can count in sequence (to the 12th multiple): 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120. (Year 1)
Silver – I know the multiplication facts,
e.g. 10 x 9 = ☐, ☐ x 10 = 40
Gold – I know the division facts,
e.g. 30 ÷ 10 = ☐, ☐ ÷ 10 = 8
1/10 of 100 = ☐
What number divided by 10 is 7?</td></tr>
<tr><td>Recall multiplication and division facts for the 2 times table.
<table><tr><th colspan="3">The TWO Times Table</th></tr><tr><th>Multiplication Facts</th><th></th><th>Division Facts</th></tr><tr><td>2 x 0 = 0</td><td>0</td><td>0 ÷ 2 = 0</td></tr><tr><td>2 x 1 = 2</td><td>1</td><td>2 ÷ 2 = 1</td></tr><tr><td>2 x 2 = 4</td><td>2</td><td>4 ÷ 2 = 2</td></tr><tr><td>2 x 3 = 6</td><td>3</td><td>6 ÷ 2 = 3</td></tr><tr><td>2 x 4 = 8</td><td>4</td><td>8 ÷ 2 = 4</td></tr><tr><td>2 x 5 = 10</td><td>5</td><td>10 ÷ 2 = 5</td></tr><tr><td>2 x 6 = 12</td><td>6</td><td>12 ÷ 2 = 6</td></tr><tr><td>2 x 7 = 14</td><td>7</td><td>14 ÷ 2 = 7</td></tr><tr><td>2 x 8 = 16</td><td>8</td><td>16 ÷ 2 = 8</td></tr><tr><td>2 x 9 = 18</td><td>9</td><td>18 ÷ 2 = 9</td></tr><tr><td>2 x 10 = 20</td><td>10</td><td>20 ÷ 2 = 10</td></tr><tr><td>2 x 11 = 22</td><td>11</td><td>22 ÷ 2 = 11</td></tr><tr><td>2 x 12 = 24</td><td>12</td><td>24 ÷ 2 = 12</td></tr></table></td><td>Bronze – I can count in sequence (to the 12th multiple): 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24 (Year 1)
Silver – I know the multiplication facts,
e.g. 2 x 11 = ☐, ☐ x 2 = 16
Gold – I know the division facts,
e.g. 12 ÷ 2 = ☐ , ☐ ÷ 2 = 9
What number divided by 2 is 8?
½ of 24 = ☐</td></tr>
<tr><td>Recall multiplication and division facts for the 5 times table.
<table><tr><th colspan="3">The FIVE Times Table</th></tr><tr><th>Multiplication Facts</th><th></th><th>Division Facts</th></tr><tr><td>5 x 0 = 0</td><td>0</td><td>0 ÷ 5 = 0</td></tr><tr><td>5 x 1 = 5</td><td>1</td><td>5 ÷ 5 = 1</td></tr><tr><td>5 x 2 = 10</td><td>2</td><td>10 ÷ 5 = 2</td></tr><tr><td>5 x 3 = 15</td><td>3</td><td>15 ÷ 5 = 3</td></tr><tr><td>5 x 4 = 20</td><td>4</td><td>20 ÷ 5 = 4</td></tr><tr><td>5 x 5 = 25</td><td>5</td><td>25 ÷ 5 = 5</td></tr><tr><td>5 x 6 = 30</td><td>6</td><td>30 ÷ 5 = 6</td></tr><tr><td>5 x 7 = 35</td><td>7</td><td>35 ÷ 5 = 7</td></tr><tr><td>5 x 8 = 40</td><td>8</td><td>40 ÷ 5 = 8</td></tr><tr><td>5 x 9 = 45</td><td>9</td><td>45 ÷ 5 = 9</td></tr><tr><td>5 x 10 = 50</td><td>10</td><td>50 ÷ 5 = 10</td></tr><tr><td>5 x 11 = 55</td><td>11</td><td>55 ÷ 5 = 11</td></tr><tr><td>5 x 12 = 60</td><td>12</td><td>60 ÷ 5 = 12</td></tr></table></td><td>Bronze – I can count in sequence (to the 12th multiple): 0, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60
(Year 1)
Silver – I know the multiplication facts,
e.g. 5 x 6 = ☐, ☐ x 5 = 15
Gold – I know the division facts,
e.g. 35 ÷ 5 = ☐ , ☐ ÷ 5 = 8
What number divided by 5 is 4?
1/5 of 30 = ☐</td></tr>
</table>

Bronze, Silver and Gold Approach to Learning Multiplication and Division Facts

<table>
<tr><th>Year</th><th>Facts to be Learnt – Instant Recall
(including knowing and using the commutative rule for x and related facts for ÷)</th><th>BSG Stages</th></tr>
<tr><td rowspan="3">Year 3</td><td colspan="2">Know and consolidate all previous objectives.</td></tr>
<tr><td>Count in 50s from zero.
0, 50, 100, 150, 200, 250, 300, 350, 400, 450, 500, 550, 600 . . .</td><td>Bronze – I can count in sequence and say how many 50s in the number by counting in 50s, e.g. on fingers.

Silver – I can multiply a single digit by 50 by counting in 50s or relating to the 5x table.
e.g. 50 x 6 = ☐, 7 x 50 = ☐

Gold – I can say how many 50s in the (multiple of 50) based on there being two 50s in every 100 or by relating to the 5x table.
e.g. 350 ÷ 50 = ☐</td></tr>
<tr><td>Recall multiplication and division facts for the 3 times table.
<table><tr><th colspan="3">The THREE Times Table</th></tr><tr><th>Multiplication Facts</th><th></th><th>Division Facts</th></tr><tr><td>3 x 0 = 0</td><td>0</td><td>0 ÷ 3 = 0</td></tr><tr><td>3 x 1 = 3</td><td>1</td><td>3 ÷ 3 = 1</td></tr><tr><td>3 x 2 = 6</td><td>2</td><td>6 ÷ 3 = 2</td></tr><tr><td>3 x 3 = 9</td><td>3</td><td>9 ÷ 3 = 3</td></tr><tr><td>3 x 4 = 12</td><td>4</td><td>12 ÷ 3 = 4</td></tr><tr><td>3 x 5 = 15</td><td>5</td><td>15 ÷ 3 = 5</td></tr><tr><td>3 x 6 = 18</td><td>6</td><td>18 ÷ 3 = 6</td></tr><tr><td>3 x 7 = 21</td><td>7</td><td>21 ÷ 3 = 7</td></tr><tr><td>3 x 8 = 24</td><td>8</td><td>24 ÷ 3 = 8</td></tr><tr><td>3 x 9 = 27</td><td>9</td><td>27 ÷ 3 = 9</td></tr><tr><td>3 x 10 = 30</td><td>10</td><td>30 ÷ 3 = 10</td></tr><tr><td>3 x 11 = 33</td><td>11</td><td>33 ÷ 3 = 11</td></tr><tr><td>3 x 12 = 36</td><td>12</td><td>36 ÷ 3 = 12</td></tr></table></td><td>Bronze – I can count in sequence (to the 12th multiple): 0, 3, 6, 9, 12, 15, 18, 21, 24, 27, 3 0 , 33, 36.

Silver – I know the multiplication facts,
e.g. 3 x 7 = ☐, ☐ x 3 = 33

Gold – I know the division facts,
e.g. 36 ÷ 3 = ☐, ☐ ÷ 3 = 4
What number divided by 3 is 12?
1/3 of 18 = ☐</td></tr>
<tr><td>Recall multiplication and division facts for the 4 times table.
<table><tr><th colspan="3">The FOUR Times Table</th></tr><tr><th>Multiplication Facts</th><th></th><th>Division Facts</th></tr><tr><td>4 x 0 = 0</td><td>0</td><td>0 ÷ 4 = 0</td></tr><tr><td>4 x 1 = 4</td><td>1</td><td>4 ÷ 4 = 1</td></tr><tr><td>4 x 2 = 8</td><td>2</td><td>8 ÷ 4 = 2</td></tr><tr><td>4 x 3 = 12</td><td>3</td><td>12 ÷ 4 = 3</td></tr><tr><td>4 x 4 = 16</td><td>4</td><td>16 ÷ 4 = 4</td></tr><tr><td>4 x 5 = 20</td><td>5</td><td>20 ÷ 4 = 5</td></tr><tr><td>4 x 6 = 24</td><td>6</td><td>24 ÷ 4 = 6</td></tr><tr><td>4 x 7 = 28</td><td>7</td><td>28 ÷ 4 = 7</td></tr><tr><td>4 x 8 = 32</td><td>8</td><td>32 ÷ 4 = 8</td></tr><tr><td>4 x 9 = 36</td><td>9</td><td>36 ÷ 4 = 9</td></tr><tr><td>4 x 10 = 40</td><td>10</td><td>40 ÷ 4 = 10</td></tr><tr><td>4 x 11 = 44</td><td>11</td><td>44 ÷ 4 = 11</td></tr><tr><td>4 x 12 = 48</td><td>12</td><td>48 ÷ 4 = 12</td></tr></table></td><td>Bronze – I can count in sequence (to the 12th multiple): 0, 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48.

Silver – I know the multiplication facts,
e.g. 6 x 4 = ☐, 4 x ☐ = 16

Gold – I know the division facts,
e.g. 12 ÷ 4 = ☐, ☐ ÷ 4 = 10
What number divided by 4 is 12?
1/4 of 36 = ☐</td></tr>
</table>

Bronze, Silver and Gold Approach to Learning Multiplication and Division Facts

<table>
<tr><th>Year</th><th>Facts to be Learnt – Instant Recall
(including knowing and using the commutative rule for x and related facts for ÷)</th><th>BSG Stages</th></tr>
<tr><td>Year 3 contd.</td><td>
Recall multiplication and division facts for the 8 times table.
<table>
<tr><th colspan="3">The EIGHT Times Table</th></tr>
<tr><th>Multiplication Facts</th><th></th><th>Division Facts</th></tr>
<tr><td>8 x 0 = 0</td><td>0</td><td>0 ÷ 8 = 0</td></tr>
<tr><td>8 x 1 = 8</td><td>1</td><td>8 ÷ 8 = 1</td></tr>
<tr><td>8 x 2 = 16</td><td>2</td><td>16 ÷ 8 = 2</td></tr>
<tr><td>8 x 3 = 24</td><td>3</td><td>24 ÷ 8 = 3</td></tr>
<tr><td>8 x 4 = 32</td><td>4</td><td>32 ÷ 8 = 4</td></tr>
<tr><td>8 x 5 = 40</td><td>5</td><td>40 ÷ 8 = 5</td></tr>
<tr><td>8 x 6 = 48</td><td>6</td><td>48 ÷ 8 = 6</td></tr>
<tr><td>8 x 7 = 56</td><td>7</td><td>56 ÷ 8 = 7</td></tr>
<tr><td>8 x 8 = 64</td><td>8</td><td>64 ÷ 8 = 8</td></tr>
<tr><td>8 x 9 = 72</td><td>9</td><td>72 ÷ 8 = 9</td></tr>
<tr><td>8 x 10 = 80</td><td>10</td><td>80 ÷ 8 = 10</td></tr>
<tr><td>8 x 11 = 88</td><td>11</td><td>88 ÷ 8 = 11</td></tr>
<tr><td>8 x 12 = 96</td><td>12</td><td>96 ÷ 8 = 12</td></tr>
</table>
</td><td>
Bronze – I can count in sequence (to the 12th multiple): 0, 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96.

Silver – I know the multiplication facts,
e.g. 8 x 9 = ☐ , 8 x ☐ = 88

Gold – I know the division facts,
e.g. 56 ÷ 8 = ☐ , ☐ ÷ 8 = 3
What number divided by 8 is 9?
1/8 of 64 = ☐
</td></tr>
<tr><td colspan="3"></td></tr>
</table>

Bronze, Silver and Gold Approach to Learning Multiplication and Division Facts

<table>
<tr><th>Year</th><th>Facts to be Learnt – Instant Recall
(including knowing and using the commutative rule for x and related facts for ÷)</th><th>BSG Stages</th></tr>
<tr><td rowspan="3">Year 4</td><td colspan="2">Know and consolidate all previous objectives.</td></tr>
<tr><td>Count in 25s.
0, 25, 50, 75, 100, 125, 150, 175, 200, 225, 250, 275, 300 . . .</td><td>Bronze – I can count in sequence.

Silver – I can say how many 25s in the number (multiple of 25) by counting in 25s, e.g. on fingers.

Gold – I can say how many 25s in the number (multiple of 25) based on there being four 25s in every 100.
e.g. 350 ÷ 25 is 4 x 3 plus 2 more so 350 ÷ 25 = 14</td></tr>
<tr><td>Recall multiplication and division facts for the 6 times table.

<table>
<tr><th colspan="3">The SIX Times Table</th></tr>
<tr><th>Multiplication Facts</th><th></th><th>Division Facts</th></tr>
<tr><td>6 x 0 = 0</td><td>0</td><td>0 ÷ 6 = 0</td></tr>
<tr><td>6 x 1 = 6</td><td>1</td><td>6 ÷ 6 = 1</td></tr>
<tr><td>6 x 2 = 12</td><td>2</td><td>12 ÷ 6 = 2</td></tr>
<tr><td>6 x 3 = 18</td><td>3</td><td>18 ÷ 6 = 3</td></tr>
<tr><td>6 x 4 = 24</td><td>4</td><td>24 ÷ 6 = 4</td></tr>
<tr><td>6 x 5 = 30</td><td>5</td><td>30 ÷ 6 = 5</td></tr>
<tr><td>6 x 6 = 36</td><td>6</td><td>36 ÷ 6 = 6</td></tr>
<tr><td>6 x 7 = 42</td><td>7</td><td>42 ÷ 6 = 7</td></tr>
<tr><td>6 x 8 = 48</td><td>8</td><td>48 ÷ 6 = 8</td></tr>
<tr><td>6 x 9 = 54</td><td>9</td><td>54 ÷ 6 = 9</td></tr>
<tr><td>6 x 10 = 60</td><td>10</td><td>60 ÷ 6 = 10</td></tr>
<tr><td>6 x 11 = 66</td><td>11</td><td>66 ÷ 6 = 11</td></tr>
<tr><td>6 x 12 = 72</td><td>12</td><td>72 ÷ 6 = 12</td></tr>
</table>
</td><td>Bronze – I can count in sequence (to the 12th multiple): 0, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72.

Silver – I know the multiplication facts,
e.g. 6 x 7 = ☐, ☐ x 6 = 36

Gold – I know the division facts,
e.g. 54 ÷ 6 = ☐, ☐ ÷ 6 = 8
What number divided by 6 is 7?
1/6 of 72 = ☐</td></tr>
<tr><td>Recall multiplication and division facts for the 11 times table.

<table>
<tr><th colspan="3">The ELEVEN Times Table</th></tr>
<tr><th>Multiplication Facts</th><th></th><th>Division Facts</th></tr>
<tr><td>11 x 0 = 0</td><td>0</td><td>0 ÷ 11 = 0</td></tr>
<tr><td>11 x 1 = 11</td><td>1</td><td>11 ÷ 11 = 1</td></tr>
<tr><td>11 x 2 = 22</td><td>2</td><td>22 ÷ 11 = 2</td></tr>
<tr><td>11 x 3 = 33</td><td>3</td><td>33 ÷ 11 = 3</td></tr>
<tr><td>11 x 4 = 44</td><td>4</td><td>44 ÷ 11 = 4</td></tr>
<tr><td>11 x 5 = 55</td><td>5</td><td>55 ÷ 11 = 5</td></tr>
<tr><td>11 x 6 = 66</td><td>6</td><td>66 ÷ 11 = 6</td></tr>
<tr><td>11 x 7 = 77</td><td>7</td><td>77 ÷ 11 = 7</td></tr>
<tr><td>11 x 8 = 88</td><td>8</td><td>88 ÷ 11 = 8</td></tr>
<tr><td>11 x 9 = 99</td><td>9</td><td>99 ÷ 11 = 9</td></tr>
<tr><td>11 x 10 = 110</td><td>10</td><td>110 ÷ 11 = 10</td></tr>
<tr><td>11 x 11 = 121</td><td>11</td><td>121 ÷ 11 = 11</td></tr>
<tr><td>11 x 12 = 132</td><td>12</td><td>132 ÷ 11 = 12</td></tr>
</table>
</td><td>Bronze – I can count in sequence (to the 12th multiple): 0, 11, 22, 33, 44, 55, 66, 77, 88, 99, 110, 121, 132.

Silver – I know the multiplication facts,
e.g. 2 x 11 = ☐, 11 x ☐ = 121

Gold – I know the division facts,
e.g. 132 ÷ 11 = ☐, ☐ ÷ 11 = 9
What number divided by 11 is 5?
1/11 of 121 = ☐</td></tr>
</table>

Bronze, Silver and Gold Approach to Learning Multiplication and Division Facts

<table>
<tr><th>Year</th><th>Facts to be Learnt – Instant Recall
(including knowing and using the commutative rule for x and related facts for ÷)</th><th>BSG Stages</th></tr>
<tr><td rowspan="3">Year 4 contd.</td><td>

Recall multiplication and division facts for the 7 times table.

The SEVEN Times Table		
Multiplication Facts		Division Facts
7 x 0 = 0	0	0 ÷ 7 = 0
7 x 1 = 7	1	7 ÷ 7 = 1
7 x 2 = 14	2	14 ÷ 7 = 2
7 x 3 = 21	3	21 ÷ 7 = 3
7 x 4 = 28	4	28 ÷ 7 = 4
7 x 5 = 35	5	35 ÷ 7 = 5
7 x 6 = 42	6	42 ÷ 7 = 6
7 x 7 = 49	7	49 ÷ 7 = 7
7 x 8 = 56	8	56 ÷ 7 = 8
7 x 9 = 63	9	63 ÷ 7 = 9
7 x 10 = 70	10	70 ÷ 7 = 10
7 x 11 = 77	11	77 ÷ 7 = 11
7 x 12 = 84	12	84 ÷ 7 = 12

</td><td>

Bronze – I can count in sequence (to the 12th multiple): *0, 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84.*

Silver – I know the multiplication facts,
e.g. 7 x 12 = ☐ , 7 x ☐ = 49

Gold – I know the division facts,
e.g. 28 ÷ 7 = ☐ , ☐ ÷ 7 = 12
What number divided by 7 is 9?
1/7 of 49 = ☐

</td></tr>
<tr><td>

Recall multiplication and division facts for the 9 times table.

The NINE Times Table		
Multiplication Facts		Division Facts
9 x 0 = 0	0	0 ÷ 9 = 0
9 x 1 = 9	1	9 ÷ 9 = 1
9 x 2 = 18	2	18 ÷ 9 = 2
9 x 3 = 27	3	27 ÷ 9 = 3
9 x 4 = 36	4	36 ÷ 9 = 4
9 x 5 = 45	5	45 ÷ 9 = 5
9 x 6 = 54	6	54 ÷ 9 = 6
9 x 7 = 63	7	63 ÷ 9 = 7
9 x 8 = 72	8	72 ÷ 9 = 8
9 x 9 = 81	9	81 ÷ 9 = 9
9 x 10 = 90	10	90 ÷ 9 = 10
9 x 11 = 99	11	99 ÷ 9 = 11
9 x 12 = 108	12	108 ÷ 9 = 12

</td><td>

Bronze – I can count in sequence (to the 12th multiple): *0, 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99, 108.*

Silver – I know the multiplication facts,
e.g. 9 x 8 = ☐ , ☐ x 9 = 63

Gold – I know the division facts,
e.g. 36 ÷ 9 = ☐ , ☐ ÷ 9 = 8
What number divided by 9 is 12?
1/9 of 81 =

</td></tr>
<tr><td>

Recall multiplication and division facts for the 12 times table.

The TWELVE Times Table		
Multiplication Facts		Division Facts
12 x 0 = 0	0	0 ÷ 12 = 0
12 x 1 = 12	1	12 ÷ 12 = 1
12 x 2 = 24	2	24 ÷ 12 = 2
12 x 3 = 36	3	36 ÷ 12 = 3
12 x 4 = 48	4	48 ÷ 12 = 4
12 x 5 = 60	5	60 ÷ 12 = 5
12 x 6 = 72	6	72 ÷ 12 = 6
12 x 7 = 84	7	84 ÷ 12 = 7
12 x 8 = 96	8	96 ÷ 12 = 8
12 x 9 = 108	9	108 ÷ 12 = 9
12 x 10 = 120	10	120 ÷ 12 = 10
12 x 11 = 132	11	132 ÷ 12 = 11
12 x 12 = 144	12	144 ÷ 12 = 12

</td><td>

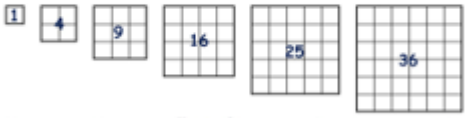
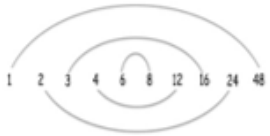
Bronze – I can count in sequence (to the 12th multiple): *0, 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144.*

Silver – I know the multiplication facts,
e.g. 3 x 12 = ☐ , ☐ x 12 = 132

Gold – I know the division facts,
e.g. 72 ÷ 12 = ☐ , ☐ ÷ 12 = 9
What number divided by 12 is 12?
1/12 of 48 =

</td></tr>
</table>

Bronze, Silver and Gold Approach to Learning Multiplication and Division Facts

Year	Facts to be Learnt – Instant Recall (including knowing and using the commutative rule for x and related facts for ÷)	BSG Stages
Year 5	**Know and consolidate all previous objectives.**	
	Recall prime numbers up to 19. 2, 3, 5, 7, 11, 13, 17, 19 . . . **Prime Numbers** **Prime numbers are numbers with only two factors. A prime number only has one and itself as factors.** One is not a prime number as it only has one factor. Two is the only even prime number. 	**Bronze** – I know the definition of a prime number and can recall prime numbers up to 19: *2, 3, 5, 7, 11, 13, 17, and 19* 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, and 144. 9 squared (9^2) is 81; the square root of 64 ($\sqrt{64}$) is 8.
	Recognise and use square numbers. 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144. **Square Numbers** When a number is multiplied by itself you get a square number. It is called a square number because you can show its factors as a square array.  You can write a small 2 after a number to square it. $1^2 = 1$ $2^2 = 4$ $3^2 = 9$ $4^2 = 16$	**Silver** – I can recall square numbers and square roots up to 12 x 12:
	Find all factor pairs of a number and common factors of two numbers. e.g. factor pairs of 24 are 1 and 24, 2 and 12, 3 and 8, 4 and 6. e.g. common factors of 15 and 20 are 1 and 5. **Factors** **Factors are the numbers you multiply together to get another number.** They divide exactly into a number, e.g. 5 is a factor of 20. The factors of 48 are listed here. The factor pairs have been linked. 	**Gold** – I can find all factor pairs of a number and common factors of two numbers, e.g. the factor pairs of 24 are 1 & 24, 2 & 12, 3 & 8, 4 & 6; common factors of 12 and 16 are 1, 2 and 4.

Bronze, Silver and Gold Approach to Learning Multiplication and Division Facts

<table>
<tr><th>Year</th><th>Facts to be Learnt – Instant Recall
(including knowing and using the commutative rule for x and related facts for ÷)</th><th>BSG Stages</th></tr>
<tr><td rowspan="3">Year 6</td><td>Know and consolidate all previous objectives.</td><td></td></tr>
<tr><td>Recall equivalences between simple fractions, decimals and percentages.

Equivalent Fractions, Decimals and Percentages
<table><tr><th>Fraction</th><th>Decimal</th><th>Percentage</th></tr><tr><td>1/10</td><td>0.1</td><td>10%</td></tr><tr><td>2/10 = 1/5</td><td>0.2</td><td>20%</td></tr><tr><td>¼</td><td>0.25</td><td>25%</td></tr><tr><td>3/10</td><td>0.3</td><td>30%</td></tr><tr><td>4/10 = 2/5</td><td>0.4</td><td>40%</td></tr><tr><td>5/10 = ½</td><td>0.5</td><td>50%</td></tr><tr><td>6/10 = 3/5</td><td>0.6</td><td>60%</td></tr><tr><td>7/10</td><td>0.7</td><td>70%</td></tr><tr><td>¾</td><td>0.75</td><td>75%</td></tr><tr><td>8/10 = 4/5</td><td>0.8</td><td>80%</td></tr><tr><td>9/10</td><td>0.9</td><td>90%</td></tr><tr><td>10/10 = 1</td><td>1.0</td><td>100%</td></tr></table></td><td>Bronze – I know equivalent fractions and percentages for the following decimal fractions: 0.1, 0.2, 0.25, 0.3, 0.4, 0.5, 0.6, 0.7, 0.75, 0.8, 0.9, 1.0</td></tr>
<tr><td>Convert measurements (length, mass, volume and time) from smaller units of measure to larger and vice-versa.

Measure Conversions
Length 10mm = 1cm, 100cm = 1m, 1000m = 1km (kilometre)
Mass 1000g = 1kg (kilogram)
Volume 1000ml = 1L (litre)
Time 60 seconds = 1 minute, 60 minutes = 1 hour, 24 hours = 1 day, 7 days = 1 week, 12 months = 1 year
▪ Convert mm to cm by dividing by 10, e.g. 50mm = 5cm
▪ Convert cm to mm by multiplying by 10, e.g. 60cm = 600mm
▪ Convert cm to m by dividing by 100, e.g. 300cm = 3m
▪ Convert m to cm by multiplying by 100, e.g. 2.5m = 250cm
▪ Convert m to km by dividing by 1000, e.g. 4500m = 4.5 km
▪ Convert km to m by multiplying by 1000, e.g. 6km = 6000m
▪ Convert g to kg by dividing by 1000, e.g. 300g = 0.3kg
▪ Convert kg to g by multiplying by 1000, e.g. 6.2kg = 6200g
▪ Convert ml to L by dividing by 1000, e.g. 2500ml = 2.5L
▪ Convert L to ml by multiplying by 1000, e.g. 4.5L = 4500ml</td><td>Silver – I can convert from a smaller unit of measure to a larger unit of measure, e.g. from m to km: 5500m = 5.5km

Gold – I can convert from a larger unit of measure to a smaller unit of measure, e.g. from kg to g: 3.75kg = 3750g</td></tr>
</table>

ADDITION: RECEPTION OVERVIEW

Number Facts

Automatically recall (without reference to rhymes, counting or other aids) **number bonds up to 5, including subtraction facts)** and some number bonds to 10 (see Y1).

1	2	3	4	5
0 + 1	0 + 2 1 + 1	0 + 3 1 + 2	0 + 4 1 + 3 2 + 2	0 + 5 1 + 4 2 + 3
1 - 0	2 – 0 2 – 1 2 - 2	3 – 0 3 – 1 3 – 2 3 - 3	4 – 0 4 – 1 4 – 2 4 – 3 4 - 4	5 – 0 5 – 4 5 – 3 5 – 2 5 - 1 5 - 5

10			
0 + 10 1 + 9 2 + 8	3 + 7 4 + 6 5 + 5	10 – 0 10 – 1 10 – 2 10 – 3 10 - 4	10 – 5 10 – 6 10 – 7 10 – 8 10 – 9 10 - 10

Bronze: addition facts,
e.g. 3 + 2 = 5 so 2 + 3 = 5

Silver: related subtraction facts,
e.g. 4 – 1 = 3 and 4 – 3 = 1

Gold: empty boxes,
e.g. 5 = □ + 2 2 + □ = 5

Vocabulary

subitise

add, addend, more, make, sum, total, altogether

equals, is the same as

one more, two more, ten more…

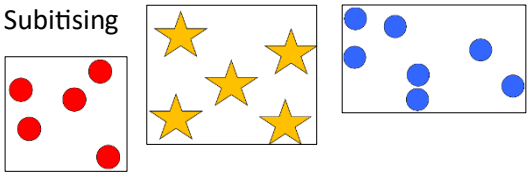
how many more to make… ? how many more is… than…?

double, combine, count on

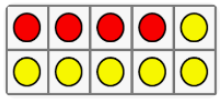
part-whole, partition

Mental Strategies/Jottings

Subitising



Pictorial representations of number facts to understand commutativity, e.g.



Combining 2 sets of objects

Adding onto a set

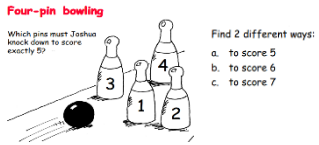
Written Method – end of year expectation

NO FORMAL WRITTEN METHODS IN EYFS

Assessment of Expected Standard

Can pupils complete the missing numbers from mental recall of number facts within 5 and some number facts to 10?
Can they find the total of two small groups of objects and talk about how they did it?
Can they say which group of objects is greater and work out how many altogether?

Challenge Opportunities



Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Mental, simple jottings or own pictorial representations, such as counting on from the largest number, e.g. within 10 or 20 with a bead frame, bead string, number line, tens-frame or fingers.	mental (or with efficient jottings) only: 37 + 56 = 80 + 13 = 93	mental: 58 + 65 = 110 + 13 = 123 written: 427 + 254 = 681 427 +254 681 1	mental: .. in my head 85 + 57 = 130 + 12 = 142 written: 6534 + 2786 = 9320 6534 + 2786 9320 111	mental: .. in my head 85 + 57 = 130 + 12 = 142 3.5 + 1.7 = 4 + 12/10 = 4 + 1.2 = 5.2 written: 31726 + 14535 31726 +14535 46361 27.46 + 85.63 27.46 +85.63 113.09 11	mental: .. in my head 85 + 67 = 140 + 12 = 152 1.75 + 4.47 = 5 + 122/100 = 5 + 1 + 22/100 = 6.22 written: 58759 + 13625 58759 +13625 72384 111 28.056 + 37.489 37.489 +28.056 65.545 111
Mental/Jottings	**Mental/Jottings**	**Mental/Jottings & Written**	**Mental/Jottings & Written**	**Mental/Jottings & Written**	**Mental/Jottings & Written**

ADDITION: YEAR 1 OVERVIEW

Number Facts

Consolidate all previous objectives and:

Derive and recall all pairs of numbers that <u>total</u> 10 ('have a sum of 10') and understand the commutative relationship, e.g. 1 + 9 = 9 + 1

Derive and recall all facts <u>within</u> 10 and understand the commutative relationship, e.g. 2 + 6 = 6 + 2

Add and subtract a multiple of 10 from a two-digit number,
e.g. 23 + 10 = 33 63 – 10 = 53

Vocabulary

subitise, add, addend, more, plus, make, sum, total, altogether

equals, is the same as

commutative

one more, two more, ten more…

combine, count on

how many more to make… ?

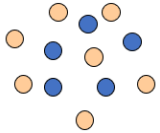
how many more is… than…?

double

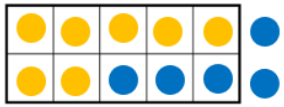
part-whole, partition

Mental Strategies/Jottings

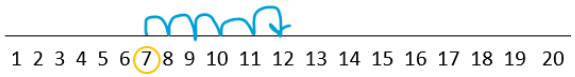
Combining 2 sets of objects



Adding onto a set



Counting on from the largest number, e.g. with a bead frame, bead string, number line, tens-frame or fingers.



Written Method – end of year expectation

NO FORMAL WRITTEN METHODS IN YEAR 1

Assessment of Expected Standard

Can pupils complete the missing numbers from mental recall of number facts within 10?

3 + 2 = ☐

6 + ☐ = 8

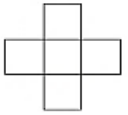
☐ + 7 = 9

4 + 3 = ☐	7 + ☐ = 9
7 – ☐ = 4	9 – ☐ = 7
5 + 2 = ☐	☐ + 3 = 9
☐ – ☐ = 2	☐ – ☐ = ☐

Do they understand the relationship between addition and subtraction and commutativity?

Greater Depth Opportunities

Write the numbers 1 to 5 in the squares so that each row and column adds up to the same number, called the 'magic number'. What is the 'magic number'?



Can you do it with 2, 3, 4, 5 and 6?

Can you do it with 4, 5, 6 ,7 and 8?

Reception	Year 2	Year 3	Year 4	Year 5	Year 6
<u>Concrete or simple pictorial representations:</u> Subitising Pictorial representations of number facts Combining 2 sets of objects Adding onto a set	<u>mental (or with efficient jottings) only:</u> 37 + 56 = 80 + 13 = 93	<u>mental:</u> 58 + 65 = 110 + 13 = 123 <u>written:</u> 427 + 254 = 681 427 +254 681 1	<u>mental:</u> .. in my head 85 + 57 = 130 + 12 = 142 <u>written:</u> 6534 + 2786 = 9320 6534 + 2786 9320 1 1 1	<u>mental:</u> .. in my head 85 + 57 = 130 + 12 = 142 3.5 + 1.7 = 4 + 12/10 = 4 + 1.2 = 5.2 <u>written:</u> 31726 + 14535 31726 + 14535 46361 27.46 + 85.63 27.46 + 85.63 113.09 1 1	<u>mental:</u> .. in my head 85 + 67 = 140 + 12 = 152 1.75 + 4.47 = 5 + 122/100 = 5 + 1 + 22/100 = 6.22 <u>written:</u> 58759 + 13625 58759 + 13625 72384 1 1 1 28.056 + 37.489 37.489 + 28.056 65.545 1 1 1
Concrete	**Mental/Jottings**	**Mental/Jottings & Written**	**Mental/Jottings & Written**	**Mental/Jottings & Written**	**Mental/Jottings & Written**

ADDITION: YEAR 2 OVERVIEW

Number Facts

Consolidate all previous objectives and:

Derive and recall all pairs of multiples of 10 with totals up to 100 and understand the commutative relationship,
e.g. 30 + 70 = 70 + 30

Derive and recall all pairs with totals to 20 ('have a sum of 20') and understand the commutative relationship,
e.g. 5 + 15 = 15 + 5

100			
0 + 100 10 + 90 20 + 80	30 + 70 40 + 60 50 + 50	100 - 0 100 – 10 100 – 20 100 – 30 100 - 40	100 – 50 100 – 60 100 – 70 100 – 80 100 – 90 100 - 100

20			
0 + 20 1 + 19 2 + 18 3 + 17 4 + 16 5 + 15 6 + 14 7 + 13 8 + 12 9 + 11 10 + 10	11 + 9 12 + 8 13 + 7 14 + 6 15 + 5 16 + 4 17 + 3 18 + 2 19 + 1 20 + 0	20 – 0 20 – 1 20 – 2 20 – 3 20 – 4 20 – 5 20 – 6 20 – 7 20 – 8 20 – 9 20 - 10	20 – 11 20 – 12 20 – 13 20 – 14 20 – 15 20 – 16 20 – 17 20 – 18 20 – 19 20 – 20

Vocabulary

addition, add, addend, more, plus, make, sum, total, altogether

equals, is the same as

commutative

one more, two more, ten more…

how many more to make… ? how many more is… than…?

combine, count on

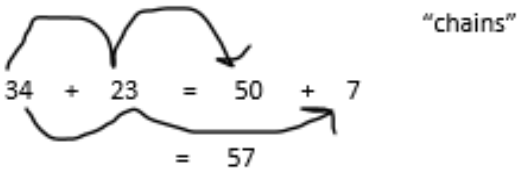
double

tens, ones, value, represent

part-whole, partition, recombine,

Mental Strategies/Jottings

General case strategy for adding a pair of 2-digit numbers:



then 37 + 56 = 80 + 13
= 93

Written Method – end of year expectation

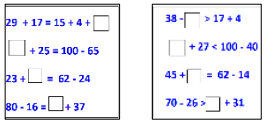
NO FORMAL WRITTEN METHODS IN YEAR 2

Assessment of Expected Standard

EXS: Add any 2 two-digit numbers using an efficient strategy, explaining their method verbally, in pictures or using apparatus (e.g. 48 + 35).

Greater Depth Opportunities

GD: Use reasoning about numbers and relationships to solve more complex problems and explain their thinking.



Reception	Year 1	Year 3	Year 4	Year 5	Year 6
Concrete or simple pictorial representations: Subitising Pictorial representations of number facts Combining 2 sets of objects Adding onto a set	Mental, simple jottings or own pictorial representations, such as counting on from the largest number, e.g. within 10 or 20 with a bead frame, bead string, number line, tens-frame or fingers.	mental: 58 + 65 = 110 + 13 = 123 written: 427 + 254 = 681 427 +254 681 1	mental: .. in my head 85 + 57 = 130 + 12 = 142 written: 6534 + 2786 = 9320 6534 + 2786 9320 1 1 1	mental: .. in my head 85 + 57 = 130 + 12 = 142 3.5 + 1.7 = 4 + 12/10 = 4 + 1.2 = 5.2 written: 31726 + 14535 31726 +14535 46361 27.46 + 85.63 27.46 +85.63 113.09 1 1	mental: .. in my head 85 + 67 = 140 + 12 = 152 1.75 + 4.47 = 5 + 122/100 = 5 + 1 + 22/100 = 6.22 written: 58759 + 13625 58759 + 13625 72384 1 1 1 28.056 + 37.489 37.489 + 28.056 65.545 1 1 1
Concrete	**Jottings**	**Mental/Jottings & Written**	**Mental/Jottings & Written**	**Mental/Jottings & Written**	**Mental/Jottings & Written**

ADDITION: YEAR 3 OVERVIEW

Number Facts — Consolidate all previous objectives and:

Derive and recall all addition and subtraction facts for each number within 15 and understand the commutative relationship, e.g. 4 + 11 = 11 + 4

Derive and recall all addition and subtraction facts for each number within 20 and understand the commutative relationship, e.g. 3 + 14 = 14 + 3

Derive and recall sums (+) and differences (-) of multiples of 10 and understand the commutative relationship, e.g. 30 + 40 = 40 + 30

Derive and recall all pairs that total 100 (complements to 100), e.g. 32 + 68 *(using the knowledge that the tens need to total 90 and the ones need to total 10)* **and the related subtraction facts.**

Vocabulary

addition, add, addend, more, make, sum, total, altogether

equals, is the same as

commutative, commutativity

complement

how many more to make… ? how many more is… than…?

inverse

double

count on

increase

partition, recombine

hundreds, tens, ones, value, represent

column

Mental Strategies/Jottings

General case strategy for mentally adding a pair of 2-digit numbers, as Y2:

58 + 65 = 110 + 13
= 123

Special case strategy of *adjusting* only if/when general strategy is secure and confidently chosen by child to solve problems:

47 + 29 = 47 + 30 – 1
= 77 – 1
= 76

Written Method – end of year expectation

Formal (column) method of addition for addition of pairs **of 3-digit numbers** when this cannot be done mentally.

427 + 254 = 681

```
 427
+254
 681
  1
```

Assessment of Expected Standard

Mark the work of these children.
What advice would you give them?
What have they done well? What do they need to improve at?

Child A	Child B	Child C
239 + 482 = 611	567 + 383 = 950	346 + 615 = 962

Greater Depth Opportunities

NCETM Teaching for Mastery:

There are six 3-digit addition calculations shown below.

a) 124 + 233 = ____
b) 644 + 172 = ____
c) 366 + 277 = ____
d) 579 + 221 = ____
e) 791 + 163 = ____
f) 567 + 233 = ____

Which calculations have no carry digits?
Which calculations have a carrying digit only once?
Which calculations have a carrying digit twice?
Which calculation has the largest answer?
Which calculation has the smallest answer?

Check that children are looking at the numbers involved, rather than doing the calculations.

Reception	Year 1	Year 2
Concrete or simple pictorial representations: Subitising Pictorial representations of number facts Combining 2 sets of objects Adding onto a set	Mental, simple jottings or own pictorial representations, such as counting on from the largest number, e.g. within 10 or 20 with a bead frame, bead string, number line, tens-frame or fingers.	mental (or with efficient jottings) only: 37 + 56 = 80 + 13 = 93
Concrete	**Jottings**	**Mental/Jottings**

Year 4		Year 5		Year 6	
mental:	written:	mental:	written:	mental:	written:
.. in my head 85 + 57 = 130 + 12 = 142	6534 + 2786 = 9320 6534 + 2786 9320 1 1 1	.. in my head 85 + 57 = 130 + 12 = 142 3.5 + 1.7 = 4 + 12 10 = 4 + 1.2 = 5.2	31726 + 14535 31726 + 14535 46361 27.46 + 85.63 27.46 + 85.63 113.09 1 1	.. in my head 85 + 67 = 140 + 12 = 152 1.75 + 4.47 = 5 + 122 100 = 5 + 1 + 22 100 = 6.22	58759 + 13625 58759 + 13625 72384 1 1 1 28.056 + 37.489 37.489 + 28.056 65.545 1 1 1
Mental/Jottings & Written		**Mental/Jottings & Written**		**Mental/Jottings & Written**	

ADDITION: YEAR 4 OVERVIEW

Number Facts

Consolidate all previous objectives and:

Derive sums and differences of pairs of multiples of 100 or 1000.

Add or subtract mentally pairs of two-digit whole numbers, e.g. 47 + 58, 91 - 35

200	300	400	500
0 + 200 100 + 100 200 + 00	0 + 300 100 + 200	0 + 400 100 + 300 200 + 200	0 + 500 100 + 400 200 + 300
200 – 0 200 – 100 200 - 200	300 – 0 300 – 100 300 – 200 300 - 300	400 – 0 400 – 100 400 – 200 400 – 300 400 - 400	500 – 0 500 – 100 500 – 200 500 – 300 500 – 400 500 - 500
600	**700**	**800**	**900**
0 + 600 100 + 500 200 + 400 300 + 300	0 + 70 100 + 600 200 + 500 300 + 400	0 + 80 100 + 700 200 + 600 300 + 500 400 + 400	0 + 900 100 + 800 200 + 700 300 + 600 400 + 500
600 – 0 600 - 100 600 – 200 600 – 300 600 – 400 600 – 500 600 - 600	700 - 0 700 – 100 700 – 200 700 – 300 700 – 400 700 – 500 700 – 600 700 - 700	800 – 0 800 – 100 800 – 200 800 – 300 800 – 400 800 – 500 800 – 600 800 – 700 800 - 800	900 – 0 900 – 100 900 – 200 900 – 300 900 – 400 900 – 500 900 – 600 900 – 700 900 – 800 900 - 900

Vocabulary

addition, add, addend, more, make, sum, total, altogether

equals, is the same as

commutative, commutativity

complement

how many more to make… ? how many more is… than…?

inverse

double

count on

increase

partition, recombine

thousands, hundreds, tens, ones, value, represent

column

decimal place

Mental Strategies/Jottings

Continue the general case strategy for *mentally* adding a pair of 2-digit numbers:

85 + 57 = 130 + 12 *... in my head*
= 142

aiming for *no* jottings here by the end of Year 4.

Special case strategy of *adjusting* only if/when general strategy is secure and confidently chosen by child to solve problems:

584 + 198 = 784 -2 *... in my head*
= 782

Written Method – end of year expectation

Formal (columnar) method of addition for addition of numbers **with 4-digits** when this cannot be done mentally .

6534 + 2786 = 9320

```
  6534
+ 2786
  9320
  111
```

Assessment of Expected Standard

NCETM Teaching for Mastery:

Decide on a mental or written strategy for each of these calculations and perform them with fluency.

- 64 + 36
- 640 + 360
- 64 + 79 + 36
- 378 + 562
- 876 + 921
- 999 + 999
- 1447 + 2362
- 1999 + 874

Greater Depth Opportunities

☐ + 3475 = 6☐24

What numbers go in the boxes?
What different answers are there?
Convince me that you have found them all – explain in words how you know.

Reception	Year 1	Year 2	Year 3
Concrete or simple pictorial representations: Subitising Pictorial representations of number facts Combining 2 sets of objects Adding onto a set	Mental, simple jottings or own pictorial representations, such as counting on from the largest number, e.g. within 10 or 20 with a bead frame, bead string, number line, tens-frame or fingers.	mental (or with efficient jottings) only: 37 + 56 = 80 + 13 = 93	mental: 58 + 65 = 110 + 13 = 123 written: 427 + 254 = 681 427 +254 681 1
Concrete	**Jottings**	**Mental/Jottings**	**Mental/Jottings & Written**

Year 5	Year 6
mental: ... in my head 85 + 57 = 130 + 12 = 142 3.5 + 1.7 = 4 + 12/10 = 4 + 1.2 = 5.2 written: 31726 + 14535 31726 +14535 46361 27.46 + 85.63 27.46 +85.63 113.09 11	mental: ... in my head 85 + 67 = 140 + 12 = 152 1.75 + 4.47 = 5 + 122/100 = 5 + 1 + 22/100 = 6.22 written: 58759 + 13625 58759 +13625 72384 1 1 1 28.056 + 37.489 37.489 +28.056 65.545 1 1 1
Mental/Jottings & Written	**Mental/Jottings & Written**

ADDITION: YEAR 5 OVERVIEW

Number Facts

Consolidate all previous number facts for instant recall and:

Derive sums and differences of decimals, using the same partitioning and recombining strategy for adding pairs of 2-digit numbers,

e.g. 2.7 + 3.9 = 5 + 1.6
= 6.6

... in my head or with jottings to 'hold' the numbers, if needed)

Vocabulary

addition, add, addend, more, make, sum, total, altogether

equals, is the same as

commutative, commutativity

complement

how many more to make… ? how many more is… than…?

inverse

double

count on

increase

partition, recombine

thousands, hundreds, tens, ones, value, represent

column

decimal place

Mental Strategies/Jottings

Continue the general case strategy for *mentally* adding a pair of 2-digit numbers:

85 + 57 = 130 + 12
= 142

... in my head

This should be done mentally *before* Year 5.

Applying the same partitioning and recombining **strategy to decimal numbers**,

e.g. 3.5 + 1.7 = 4 + 12/10
= 4 + 1.2
= 5.2

aiming for *no* jottings here by the end of Year 5.

Written Method – end of year expectation

Formal (columnar) method of addition for addition of numbers **with more than 4-digits**, (including those with up to 2 decimal places) when this cannot be done mentally.

31726 + 14535 = 46261

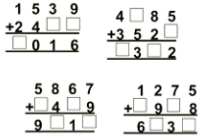
```
  31726
+ 14535
  46261
   1  1
```

27.46 + 85.63 = 113.09

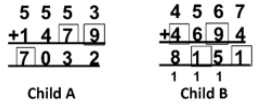
```
  27.46
+ 85.63
 113.09
  1 1
```

Assessment of Expected Standard

Work out the missing numbers and write the original calculations horizontally.



Mark these children's work. Have they calculated correctly? What advice would you give?



Although this shows 4-digit calculations, the missing numbers make them more complex to solve.

Greater Depth Opportunities

NCETM Teaching for Mastery: Sam and Tom have £67·80 between them. If Sam has £6·20 more than Tom, how much does Tom have?

Reception	Year 1	Year 2	Year 3	Year 4	Year 6
Concrete or simple pictorial representations: Subitising Pictorial representations of number facts Combining 2 sets of objects Adding onto a set	Mental, simple jottings or own pictorial representations, such as counting on from the largest number, e.g. within 10 or 20 with a bead frame, bead string, number line, tens-frame or fingers.	mental (or with efficient jottings) only: 37 + 56 = 80 + 13 = 93	mental: 58 + 65 = 110 + 13 = 123 written: 427 + 254 = 681 427 +254 681 1	mental: *.. in my head* 85 + 57 = 130 + 12 = 142 written: 6534 + 2786 = 9320 6534 + 2786 9320 1 1 1	mental: *.. in my head* 85 + 67 = 140 + 12 = 152 1.75 + 4.47 = 5 + 122/100 = 5 + 1 + 22/100 = 6.22 written: 58759 + 13625 58759 + 13625 72384 1 1 1 28.056 + 37.489 37.489 + 28.056 65.545 1 1 1
Concrete	**Jottings**	**Mental/Jottings**	**Mental/Jottings & Written**	**Mental/Jottings & Written**	**Mental/Jottings & Written**

ADDITION: YEAR 6 OVERVIEW

Number Facts

Consolidate all previous number facts for instant recall including:

Derive sums and differences of decimals, using the same partitioning and recombining strategy for adding pairs of 2-digit numbers,

e.g. 9.38 + 2.46 = 11 + 84/100 = 11.84

OR 9.38 + 2.46 = 11 + 0.7 + 0.14 = 11.84

and apply all KS2 +/- facts/strategies within problem solving contexts, e.g. measure; use these instantly known facts instead of inefficient vertical written methods.

Vocabulary

addition, add, addend, more, make, sum, total, altogether

equals, is the same as

commutative, commutativity

complement

how many more to make… ? how many more is… than…?

inverse

double

count on

increase

partition, recombine

thousands, hundreds, tens, ones, value, represent

column

decimal place

Mental Strategies/Jottings

Continue the general case strategy for *mentally* adding a pair of 2-digit numbers:

85 + 67 = 140 + 12 = 152 *… in my head*

This should be done mentally by the end of Year 4.

Applying the same partitioning and recombining **strategy to decimal numbers**,

e.g. 1.75 + 4.47 = 5 + 122/100 = 5 + 1 + 22/100 = 6.22

aiming for *minimal* or *no* jottings here by the end of Year 6.

Written Method – end of year expectation

Formal (columnar) method of addition for addition of numbers **with more than 4-digits**, (including those with up to 3 decimal places) when this cannot be done mentally.

58759 + 13625 = 72384

```
 58759
+13625
 72384
 1 1  1
```

28.056 + 37.489 = 113.09

```
 37.489
+28.056
 65.545
 1  1 1
```

Assessment of Expected Standard

NCETM Teaching for Mastery:

Calculate 36·2 + 19·8
- with a formal written column method
- with a mental method, explaining your reasoning.

Choose digits to go in the empty boxes to make these number sentences true.

14 781 – 6 ☐ 53 = 8528

23·12 + 22· ☐ = 45·23

Greater Depth Opportunities

NCETM Teaching for Mastery:

Can you use five of the digits 1 to 9 to make this number sentence true?

☐☐·☐+☐·☐ = 31·7

Can you find other sets of five of the digits 1 to 9 that make the sentence true?

NRICH: *Alphabet Soup*

Two numbers have a difference of 2·38. What could the numbers be if:
- the two numbers add up to 6?
- one of the numbers is three times as big as the other number?

Two numbers have a difference of 2·3. To the nearest 10, they are both 10. What could the numbers be?

Reception	Year 1	Year 2	Year 3	Year 4	Year 5
Concrete or simple pictorial representations: Subitising Pictorial representations of number facts Combining 2 sets of objects Adding onto a set	Mental, simple jottings or own pictorial representations, such as counting on from the largest number, e.g. within 10 or 20 with a bead frame, bead string, number line, tens-frame or fingers.	mental (or with efficient jottings) only: 37 + 56 = 80 + 13 = 93	mental: 58 + 65 = 110 + 13 = 123 written: 427 + 254 = 681 427 +254 681	mental: .. in my head 85 + 57 = 130 + 12 = 142 written: 6534 + 2786 = 9320 6534 + 2786 9320	mental: .. in my head 85 + 57 = 130 + 12 = 142 3.5 + 1.7 = 4 + 12/10 = 4 + 1.2 = 5.2 written: 31726 + 14535 31726 +14535 46361 27.46 + 85.63 27.46 +85.63 113.09
Concrete	**Jottings**	**Mental/Jottings**	**Mental/Jottings & Written**	**Mental/Jottings & Written**	**Mental/Jottings & Written**

SUBTRACTION: RECEPTION OVERVIEW

Number Facts

Automatically recall (without reference to rhymes, counting or other aids) **number bonds up to 5, including subtraction facts)** and some number bonds to 10 (see Y1).

1	2	3	4	5
0 + 1	0 + 2 1 + 1	0 + 3 1 + 2	0 + 4 1 + 3 2 + 2	0 + 5 1 + 4 2 + 3
1 - 0	2 – 0 2 – 1 2 - 2	3 – 0 3 – 1 3 – 2 3 - 3	4 – 0 4 – 1 4 – 2 4 – 3 4 - 4	5 – 0 5 – 4 5 – 3 5 – 2 5 - 1 5 - 5

10			
0 + 10 1 + 9 2 + 8	3 + 7 4 + 6 5 + 5	10 – 0 10 – 1 10 – 2 10 – 3 10 - 4	10 – 5 10 – 6 10 – 7 10 – 8 10 – 9 10 - 10

Bronze: addition facts,
e.g. 3 + 2 = 5 so 2 + 3 = 5

Silver: related subtraction facts,
e.g. 4 – 1 = 3 and 4 – 3 = 1

Gold: empty boxes,
e.g. 5 = □ + 2 2 + □ = 5

Vocabulary

subitise

take (away), leave

one less, two less… ten less…

how many fewer is… than…? how many are left/left over? how many have gone?

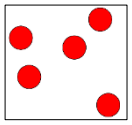
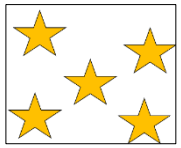
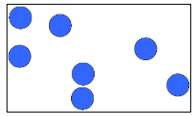
equals, is the same as

minuend, subtrahend

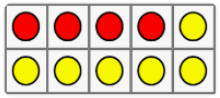
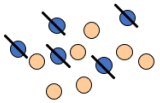
difference, difference between

Mental Strategies/Jottings

Subitising

Pictorial representations of number facts to understand commutativity, e.g.

Removing items from a set (*take away*)

Written Method – end of year expectation

NO FORMAL WRITTEN METHODS IN EYFS

Assessment of Expected Standard

Can pupils complete the missing numbers from mental recall of number facts within 5 and some number facts to 10?

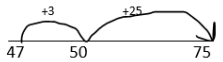
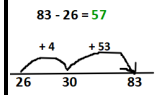
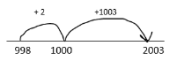
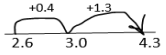
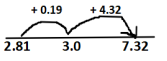
Can they take away an amount from a small groups of objects and work out how many they have left?

Can they say which group of objects has fewest and work out how many less it has than the other?

Greater Depth Opportunities

Use Cuisenaire Rods:

The dark green rod (6) is one more than the yellow rod (5).
Can you find other pairs of rods with a difference of one?
Now find pairs of rods with a difference of 2. How many are there?
Now find pairs of rods with a difference of 3. How many do you think there will be? Why?
What about a difference of 4 or 5?
Can you talk about the pattern you're finding?

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Mental, simple jottings or own pictorial representation to count back to take away or count on to find the difference, e.g. within 10 or 20 with a bead frame, bead string, number line, tens-frame or fingers.	mental (or with efficient jottings) only: 75 – 47 = 28 	mental: 83 - 26 = 57  written: 635 - 379 = 256 6 3 5 - 3 7 9 2 5 6	mental: 2003 – 998 = 1005  written: 8157 - 1678 = 6479 8 1 5 7 - 1 6 7 8 6 4 7 9	mental: 4.3 – 2.6 = 1.7  written: 41535 - 24386 = 17148 4 1 5 3 5 - 2 4 3 8 6 1 7 1 4 9	mental: £7.32 - £2.81 = £4.51  written: 63.512 - 37.843 = 25.669 6 3 . 5 1 2 - 3 7 . 8 4 3 2 5 . 6 6 9
Mental/Jottings	**Mental/Jottings**	**Mental/Jottings & Written**	**Mental/Jottings & Written**	**Mental/Jottings & Written**	**Mental/Jottings & Written**

SUBTRACTION: YEAR 1 OVERVIEW

Number Facts

Consolidate all previous objectives and:

Derive and recall all pairs of numbers that total 10 ('have a sum of 10') and understand the commutative relationship, e.g. 1 + 9 = 9 + 1 **and the related subtraction facts.**

Derive and recall all facts within 10 and understand the commutative and inverse relationship **and the related subtraction facts,**
e.g. 8 – 2 = 6 and 8 – 6 = 2

Add and subtract a multiple of 10 from a two-digit number,
e.g. 23 + 10 = 33 63 – 10 = 43

Vocabulary

take away, minus, subtract

one less, two less… ten less…

how many fewer is… than…? how many are left/left over? how many have gone?

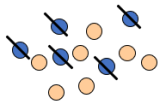
equals, is the same as

minuend, subtrahend

difference, difference between

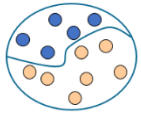
Mental Strategies/Jottings

Removing items from a set (*take away*)



Comparing two sets (*finding the difference***),** e.g. with a tens frame, bead frame or sticks of cubes.

Using known number facts to see related calculations



Written Method – end of year expectation

NO FORMAL WRITTEN METHODS IN YEAR 1

Assessment of Expected Standard

Can pupils complete the missing numbers from mental recall of number facts within 10?

9 = □ + 7

7 - □ = 4

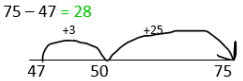
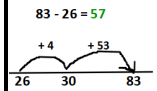
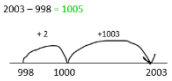
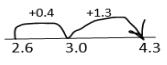
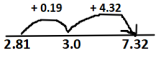
6 = □ - 7

4 + 3 = □	7 + □ = 9
7 – □ = 4	9 – □ = 7
5 + 2 = □	□ + 3 = 9
□ – □ = 2	□ – □ = □

Do they understand the relationship between addition and subtraction and commutativity?

Greater Depth Opportunities

NCETM Teaching for Mastery:
I'm thinking of a number. I've added 8 and the answer is 12. What number was I thinking of? Explain how you know.
I know that 10 take away 7 is 3. How can I find 12 - 7?

Reception	Year 2	Year 3	Year 4	Year 5	Year 6
Concrete or simple pictorial representations: Subitising Pictorial representations of number facts Removing items from a set	mental (or with efficient jottings) only: 75 – 47 = 28 	mental: 83 - 26 = 57  written: 635 - 379 = 256 635 - 379 256	mental: 2003 – 998 = 1005  written: 8157 - 1678 = 6479 8157 - 1678 6479	mental: 4.3 – 2.6 = 1.7  written: 41535 - 24386 = 17148 41535 -24386 17149	mental: £7.32 - £2.81 = £4.51  written: 63.512 - 37.843 = 25.669 63.512 - 37.843 25.669
Concrete	**Mental/Jottings**	**Mental/Jottings & Written**	**Mental/Jottings & Written**	**Mental/Jottings & Written**	**Mental/Jottings & Written**

SUBTRACTION: YEAR 2 OVERVIEW

Number Facts

Consolidate all previous objectives and:

Derive and recall all pairs of multiples of 10 with totals up to 100 and understand the commutative relationship **and the related subtraction facts,**
e.g. 100 – 70 = 30; 100 – 30 = 70

Derive and recall all pairs with totals to 20 ('have a sum of 20') and understand the commutative relationship **and the related subtraction facts,**
e.g. 20 – 15 = 5; 20 – 5 = 15

100			
0 + 100 10 + 90 20 + 80	30 + 70 40 + 60 50 + 50	100 - 0 100 – 10 100 – 20 100 – 30 100 - 40	100 – 50 100 – 60 100 – 70 100 – 80 100 – 90 100 - 100

20			
0 + 20 1 + 19 2 + 18 3 + 17 4 + 16 5 + 15 6 + 14 7 + 13 8 + 12 9 + 11 10 + 10	11 + 9 12 + 8 13 + 7 14 + 6 15 + 5 16 + 4 17 + 3 18 + 2 19 + 1 20 + 0	20 – 0 20 – 1 20 – 2 20 – 3 20 – 4 20 – 5 20 – 6 20 – 7 20 – 8 20 – 9 20 - 10	20 – 11 20 – 12 20 – 13 20 – 14 20 – 15 20 – 16 20 – 17 20 – 18 20 – 19 20 – 20

Vocabulary

take away, minus, subtract

ten less, one hundred less …

how many fewer is… than…? how many are left/left over? how many have gone?

equals, is the same as

minuend, subtrahend

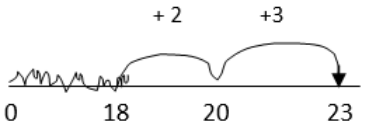
difference, difference between

half, halve

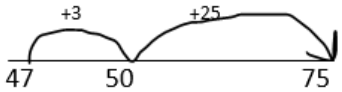
Mental Strategies/Jottings

Find the difference by counting on, using number facts and place value

23 – 18 = 5



then 75 – 47 = 28



Written Method – end of year expectation

NO FORMAL WRITTEN METHODS IN YEAR 2

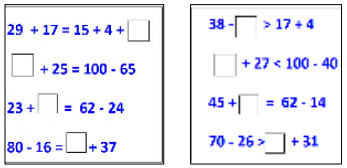
Assessment of Expected Standard

EXS: Add and subtract any 2 two-digit numbers using an efficient strategy, explaining their method verbally, in pictures or using apparatus (e.g.72 – 17).

NB: Aiming for a maximum of 3 jumps but 2 jumps is most efficient.

Greater Depth Opportunities

GD: Use reasoning about numbers and relationships to solve more complex problems and explain their thinking.



Reception	Year 1	Year 3	Year 4	Year 5	Year 6
Concrete or simple pictorial representations: Subitising Pictorial representations of number facts Removing items from a set	Mental, simple jottings or own pictorial representation to count back to take away or count on to find the difference, e.g. within 10 or 20 with a bead frame, bead string, number line, tens-frame or fingers.	mental: 83 - 26 = 57 written: 635 - 379 = 256	mental: 2003 – 998 = 1005 written: 8157 - 1678 = 6479	mental: 4.3 – 2.6 = 1.7 written: 41535 - 24386 = 17148	mental: £7.32 - £2.81 = £4.51 written: 63.512 - 37.843 = 25.669
Concrete	Jottings	Mental/Jottings & Written	Mental/Jottings & Written	Mental/Jottings & Written	Mental/Jottings & Written

SUBTRACTION: YEAR 3 OVERVIEW

Number Facts — Consolidate all previous objectives and:

Derive and recall all addition and subtraction facts for each number within 15 and understand the commutative relationship **and the related subtraction facts, e**.g. 14 – 6 = 8; 14 – 8 = 6

Derive and recall all addition and subtraction facts for each number within 20 and understand the commutative relationship **and the related subtraction facts,** e.g. 19 – 4 = 15; 19 – 15 = 4

Derive and recall sums (+) and differences (-) of multiples of 10 and understand the commutative relationship **and the related subtraction facts.** e.g. 70 – 40 = 30; 70 – 30 = 40

Derive and recall all pairs that total 100 (complements to 100), e.g. 32 + 68 *(using the knowledge that the tens need to total 90 and the ones need to total 10)* **and the related subtraction facts.**

Vocabulary

take away, minus, subtract

ten less, one hundred less …

how many fewer is… than…? how many are left/left over? how many have gone?

equals, is the same as

difference, difference between

minuend, subtrahend

inverse

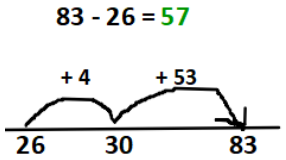
half, halve

decrease

repartition, exchange

Mental Strategies/Jottings

General case strategy for mentally subtracting a pair of 2-digit numbers, as Y2:



Special case strategy of *adjusting* only if/when general strategy is secure and confidently chosen by child to solve problems:

85 - 19 = 85 - 20 + 1

… in my head

= 65 + 1

= 66

Written Method – end of year expectation

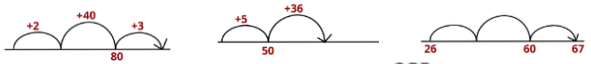
Formal (column) method of subtraction for subtraction **of pairs of 3-digit numbers** when this cannot be done mentally.

635 - 379 = 256

$$\begin{array}{r} {}^{5}\not{6}\,{}^{12}\not{3}\,{}^{1}\not{5} \\ -\ 3\ 7\ 9 \\ \hline 2\ 5\ 6 \end{array}$$

Assessment of Expected Standard

What is the subtraction calculation for each number line? Could it be more efficient? How?



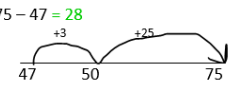
Sam has completed this calculation. What advice would you give him?

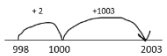
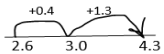
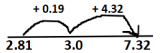
$$\begin{array}{r} 355 \\ -\ 247 \\ \hline 112 \end{array}$$

Greater Depth Opportunities

Complete the missing digits and write the calculations horizontally. Can you teach someone else to solve these problems?

$$\begin{array}{r} 8\square 1 \\ -\ 5\ 0\square \\ \hline 3\ 2\ 3 \end{array} \qquad \begin{array}{r} 5\square 3 \\ -\ \square\ 8\ 6 \\ \hline 5\ 7 \end{array} \qquad \begin{array}{r} \square\ 7\ 7 \\ -\ 7\ 2\square \\ \hline 2\ 4\ 9 \end{array}$$

Reception	Year 1	Year 2
Concrete or simple pictorial representations: Subitising Pictorial representations of number facts Removing items from a set	Mental, simple jottings or own pictorial representation to count back to take away or count on to find the difference, e.g. within 10 or 20 with a bead frame, bead string, number line, tens-frame or fingers.	mental (or with efficient jottings) only: 75 – 47 = 28 
Concrete	**Jottings**	**Mental/Jottings**

Year 4	Year 5	Year 6
mental: 2003 – 998 = 1005  	written: 8157 - 1678 = 6479 $\begin{array}{r} 8\ {}^{15}1\ {}^{14}5\ {}^{13}7 \\ -\ 1\ 6\ 7\ 8 \\ \hline 6\ 4\ 7\ 9 \end{array}$	mental: 4.3 – 2.6 = 1.7  	written: 41535 - 24386 = 17148 $\begin{array}{r} {}^{3}4\ {}^{1}1\ {}^{4}5\ {}^{12}3\ {}^{5}5 \\ -\ 2\ 4\ 3\ 8\ 6 \\ \hline 1\ 7\ 1\ 4\ 9 \end{array}$	mental: £7.32 - £2.81 = £4.51  	written: 63.512 - 37.843 = 25.669 $\begin{array}{r} {}^{5}6\ {}^{12}3\ {}^{1}.\ {}^{14}5\ {}^{10}1\ {}^{1}2 \\ -\ 3\ 7\ .\ 8\ 4\ 3 \\ \hline 2\ 5\ .\ 6\ 6\ 9 \end{array}$
Mental/Jottings & Written	**Mental/Jottings & Written**	**Mental/Jottings & Written**

SUBTRACTION: YEAR 4 OVERVIEW

Number Facts

Consolidate all previous objectives and:

Derive sums and differences of pairs of multiples of 100 or 1000,

Add or subtract mentally pairs of two-digit whole numbers,

e.g. 47 + 58, 91 - 35

200	300	400	500
0 + 200 100 + 100 200 + 00	0 + 300 100 + 200	0 + 400 100 + 300 200 + 200	0 + 500 100 + 400 200 + 300
200 – 0 200 – 100 200 - 200	300 – 0 300 – 100 300 – 200 300 - 300	400 – 0 400 – 100 400 – 200 400 – 300 400 - 400	500 – 0 500 – 100 500 – 200 500 – 300 500 – 400 500 - 500
600	**700**	**800**	**900**
0 + 600 100 + 500 200 + 400 300 + 300	0 + 70 100 + 600 200 + 500 300 + 400	0 + 80 100 + 700 200 + 600 300 + 500 400 + 400	0 + 900 100 + 800 200 + 700 300 + 600 400 + 500
600 – 0 600 - 100 600 – 200 600 – 300 600 – 400 600 – 500 600 - 600	700 - 0 700 – 100 700 – 200 700 – 300 700 – 400 700 – 500 700 – 600 700 - 700	800 – 0 800 – 100 800 – 200 800 – 300 800 – 400 800 – 500 800 – 600 800 – 700 800 - 800	900 – 0 900 – 100 900 – 200 900 – 300 900 – 400 900 – 500 900 – 600 900 – 700 900 – 800 900 - 900

Vocabulary

take away, minus, subtract

ten less, one hundred less …

how many fewer is… than…? how many are left/left over? how many have gone?

equals, is the same as

difference, difference between

minuend, subtrahend

inverse

half, halve

decrease

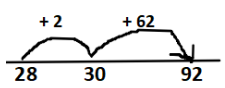
repartition, exchange

tens/hundreds/thousands boundary

Mental Strategies/Jottings

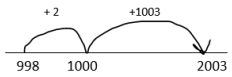
Continue the general case strategy for mentally subtracting a pair of 2-digit numbers and applying to larger numbers:

92 - 28 = 64



aiming for *no* jottings for these by the end of Year 4.

2003 – 998 = 1005



Special case strategy of *adjusting* only if/when general strategy is secure and confidently chosen by child to solve problems.

… in my head

584 - 298 = 284 + 2
= 286

Written Method – end of year expectation

Formal (columnar) method of subtraction for subtraction **of numbers with 4-digits** when this cannot be done mentally .

8157 - 1678 = 6479

7 10 14 1
~~8~~ ~~1~~ ~~5~~ ~~7~~
- 1 6 7 8
6 4 7 9

Assessment of Expected Standard

Decide on a mental or written strategy for each of these calculations and perform them with fluency.

- 72 – 35
- 680 – 240
- 8613 – 6378
- 7162 – 5475
- 91 – 32
- 924 – 799
- 2567 - 1425
- 853 – 242
- 3004 – 1998
- 6104 - 3582

Greater Depth Opportunities

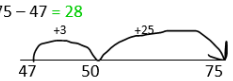
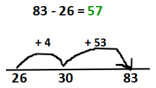
NCETM Teaching for Mastery:

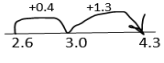
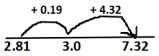
Fill in the missing digits. Explain your strategies.

1 ☐ 5 ☐ + 300 = 1557

5 ☐ 28 – 44 ☐ = 4788

☐ ☐ ☐ 0 – 2468 = 5092

Reception	Year 1	Year 2	Year 3
Concrete or simple pictorial representations: Subitising Pictorial representations of number facts Removing items from a set	Mental, simple jottings or own pictorial representation to count back to take away or count on to find the difference, e.g. within 10 or 20 with a bead frame, bead string, number line, tens-frame or fingers.	mental (or with efficient jottings) only: 75 – 47 = 28  	mental: 83 - 26 = 57  written: 635 - 379 = 256 5 12 1 6 3 5 - 3 7 9 2 5 6
Concrete	**Jottings**	**Mental/Jottings**	**Mental/Jottings & Written**

Year 5	Year 6
mental: 4.3 – 2.6 = 1.7  written: 41535 - 24386 = 17148 3 11 4 12 15 4 1 5 3 5 - 2 4 3 8 6 1 7 1 4 9	mental: £7.32 - £2.81 = £4.51  written: 63.512 - 37.843 = 25.669 5 12 14 12 6 3 . 5 1 2 - 3 7 . 8 4 3 2 5 . 6 6 9
Mental/Jottings & Written	**Mental/Jottings & Written**

SUBTRACTION: YEAR 5 OVERVIEW

Number Facts

Consolidate all previous objectives and:

Derive sums and differences of decimals, using the same partitioning and recombining strategy for adding pairs of 2-digit numbers,

e.g. 2.7 + 3.9 = 5 + 1.6
= 6.6

Vocabulary

take away, minus, subtract

ten less, one hundred less …

how many fewer is… than…? how many are left/left over? how many have gone?

equals, is the same as

difference, difference between

minuend, subtrahend inverse

half, halve

decrease

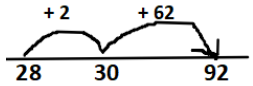
repartition, exchange

tens/hundreds/thousands boundary

Mental Strategies/Jottings

Continue the general case strategy for mentally subtracting a pair of 2-digit numbers and applying to larger numbers:

92 - 28 = 64

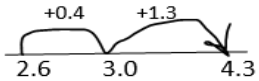


This should be done mentally *before* Year 5 and then apply to larger numbers in Year 5.

… in my head

Applying the same strategy of counting up to find the difference, using known facts and place value **to decimal numbers,** e.g.

4.3 – 2.6 = 1.7



aiming for *no* jottings here by the end of Year 5.

Written Method – end of year expectation

Formal (columnar) method of subtraction for subtraction of numbers **with more than 4-digits,** (including those with up to 2 decimal places) <u>when this cannot be done mentally.</u>

41535 - 24386 = 17148

$$\begin{array}{r} {}^{3}\cancel{4}\,{}^{1}\cancel{1}\,{}^{4}\cancel{5}\,{}^{12}\cancel{3}\,{}^{1}\cancel{5} \\ -\,2\,4\,3\,8\,6 \\ \hline 1\,7\,1\,4\,9 \end{array}$$

63.51 - 37.84 = 25.67

$$\begin{array}{r} {}^{5}\cancel{6}\,{}^{12}\cancel{3}.\,{}^{14}\cancel{5}\,{}^{1}1 \\ -\,3\,7.\,8\,4 \\ \hline 2\,5.\,6\,7 \end{array}$$

Assessment of Expected Standard

NCETM Teaching for Mastery:

Solve these calculations using a column method. Complete the missing numbers in the horizontal calculations.

3254 + ☐ = 7999

2431 = ☐ – 3456

6373 – ☐ = 3581

6719 = ☐ – 4562

Greater Depth Opportunities

NCETM Teaching for Mastery:

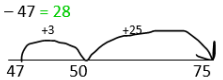
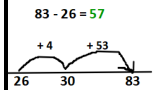
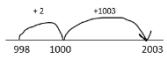
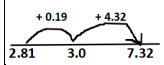
True or False?

- 3999 – 2999 = 4000 – 3000
- 3999 – 2999 = 3000 – 2000
- 2741 – 1263 = 2742 – 1264
- 2741 + 1263 = 2742 + 1264
- 2741 – 1263 = 2731 – 1253
- 2741 – 1263 = 2742 – 1252

Explain your reasoning.

Using this number statement, 5222 – 3111 = 5223 – 3112 write three more pairs of equivalent calculations.

Pupils should not calculate the answer to these questions but should look at the structure and relationships between the numbers.

Reception	Year 1	Year 2	Year 3	Year 4	Year 6
<u>Concrete or simple pictorial representations:</u> Subitising Pictorial representations of number facts Removing items from a set	<u>Mental, simple jottings or own pictorial representation</u> to count back to take away or count on to find the difference, e.g. within 10 or 20 with a bead frame, bead string, number line, tens-frame or fingers.	<u>mental (or with efficient jottings) only:</u> 75 – 47 = 28 	<u>mental:</u> 83 - 26 = 57  <u>written:</u> 635 - 379 = 256 635 - 379 256	<u>mental:</u> 2003 – 998 = 1005  <u>written:</u> 8157 - 1678 = 6479 8157 - 1678 6479	<u>mental:</u> £7.32 - £2.81 = £4.51  <u>written:</u> 63.512 - 37.843 = 25.669 63.512 - 37.843 25.669
Concrete	**Jottings**	**Mental/Jottings**	**Mental/Jottings & Written**	**Mental/Jottings & Written**	**Mental/Jottings & Written**

SUBTRACTION: YEAR 6 OVERVIEW

Number Facts

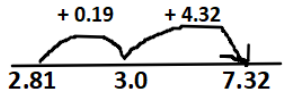
Consolidate all previous number facts for instant recall including:

Derive sums and differences of decimals, using the same strategy for subtracting pairs of 2-digit numbers by counting up to find the difference,
e.g. 9.38 - 11.52

... in my head or with jottings to 'hold' the numbers, if needed)

and apply all KS2 +/- facts/strategies within problem solving contexts, e.g. measure; use these instantly known facts instead of inefficient vertical written methods.

£7.32 - £2.81 = £4.51



Vocabulary

take away, minus, subtract

ten less, one hundred less …

how many fewer is… than…? how many are left/left over? how many have gone?

equals, is the same as

difference, difference between

minuend, subtrahend inverse

half, halve

decrease

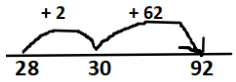
repartition, exchange

tens/hundreds/thousands boundary

Mental Strategies/Jottings

Continue the general case strategy for mentally subtracting a pair of 2-digit numbers and applying to larger numbers:

92 - 28 = 64



... in my head

This should be done mentally by the end of Year 4.

Applying the same strategy of counting up to find the difference, using known facts and place value **to decimal numbers and larger numbers** (see Year 5), **aiming for *minimal* or *no* jottings** here by the end of Year 6.

Written Method – end of year expectation

Formal (columnar) method of subtraction for subtraction of numbers **with more than 4-digits,** (including those with up to 3 decimal places) when this cannot be done mentally.

41535 - 24386 = 17148

$$\begin{array}{r} {}^{3}4\,{}^{1}1\,{}^{4}5\,{}^{12}3\,{}^{1}5 \\ -\,2\,4\,3\,8\,6 \\ \hline 1\,7\,1\,4\,9 \end{array}$$

63.512 - 37.843 = 25.669

$$\begin{array}{r} {}^{5}6\,{}^{12}3\,{}^{14}.\,{}^{10}5\,{}^{1}1\,2 \\ -\,3\,7.\,8\,4\,3 \\ \hline 2\,5.\,6\,6\,9 \end{array}$$

Assessment of Expected Standard

Calculate 8.123 – 6. 989

- with a column method
- with a mental method, explaining your reasoning.

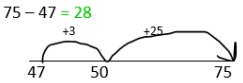
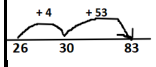
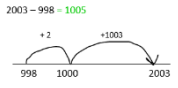
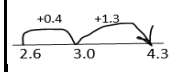
Greater Depth Opportunities

NCETM Teaching for Mastery:

Two numbers have a difference of 2·38. What could the numbers be if:

- the two numbers add up to 6?
- one of the numbers is three times as big as the other number?

Two numbers have a difference of 2·3. To the nearest 10, they are both 10. What could the numbers be?

Reception	Year 1	Year 2	Year 3	Year 4	Year 5
Concrete or simple pictorial representations: Subitising Pictorial representations of number facts Removing items from a set	Mental, simple jottings or own pictorial representation to count back to take away or count on to find the difference, e.g. within 10 or 20 with a bead frame, bead string, number line, tens-frame or fingers.	mental (or with efficient jottings) only: 75 – 47 = 28  	mental: 83 - 26 = 57  written: 635 - 379 = 256 $\begin{array}{r} {}^{5}6\,{}^{12}3\,{}^{1}5 \\ -\,3\,7\,9 \\ \hline 2\,5\,6 \end{array}$	mental:  written: 8157 - 1678 = 6479 $\begin{array}{r} {}^{7}8\,{}^{10}1\,{}^{14}5\,{}^{1}7 \\ -\,1\,6\,7\,8 \\ \hline 6\,4\,7\,9 \end{array}$	mental: 4.3 – 2.6 = 1.7  written: 41535 - 24386 = 17148 $\begin{array}{r} {}^{3}4\,{}^{1}1\,{}^{4}5\,{}^{12}3\,{}^{1}5 \\ -\,2\,4\,3\,8\,6 \\ \hline 1\,7\,1\,4\,9 \end{array}$
Concrete	**Jottings**	**Mental/Jottings**	**Mental/Jottings & Written**	**Mental/Jottings & Written**	**Mental/Jottings & Written**

MULTIPLICATION: RECEPTION OVERVIEW

Number Facts

Automatically recall (without reference to rhymes, counting or other aids) . . . **double facts** at least up to 5 + 5 = 10, e.g. double 4 is 8

Doubling Facts to 10					
0 + 0 = 0	1 + 1 = 2	2 + 2 = 4	3 + 3 = 6	4 + 4 = 8	5 + 5 = 10
double 0 is 0	double 1 is 2	double 2 is 4	double 3 is 6	double 4 is 8	double 5 is 10

Extend to Halving Facts Within 10				
half of 2 is 1	half of 4 is 2	half of 6 is 3	half of 8 is 4	half of 10 is 5

Vocabulary

equal groups of …, grouping

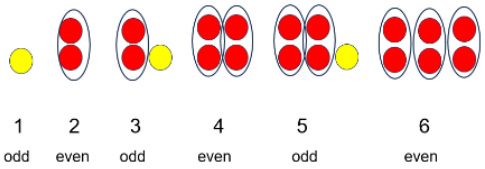
odd, even

double, doubling

Mental Strategies/Jottings

Explore and **represent patterns within numbers up to 10, including evens and odds**, double facts and how quantities can be distributed equally.

e.g. odds and evens 1, 2, 3, 4, 5, 6



Written Method – end of year expectation

NO FORMAL WRITTEN METHODS IN EYFS

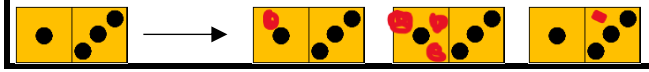
Assessment of Expected Standard

Can pupils use manipulatives to show and talk about a pattern of doubles and halves, e.g. Cuisenaire Rods, multilink cubes in 2 colours?

Can they recall doubling facts up to 5 + 5?

Greater Depth Opportunities

Can pupils draw dots on dominoes showing even numbers so that they show an odd number? Can they do it in different ways? Can they talk about what they notice?



Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
practical and pictorial: Solve problems using concrete objects, pictorial representations, e.g. arrays and equal grouping.	mental only: 42 x 2 (or double 42) = 84 x2 (or d) x2 (or d) 80 + 4	mental: 32 x 5 = 160 x10 320 ÷2 160 63 x 8 = 504 x8 x8 480 + 24 written: 63 x 8 = 504 63 x 8 504 2	mental: 124 x 5 = 620 x10 1240 ½ 620 79 x 6 = 420 + 54 = 474 written: 236 x 7 = 1652 236 x 7 1652 24	mental: 24 x 12 = 288 x 10 x 2 (or d) 240 + 48 written: 4516 x 37 = 167,092 4516 x 37 31612 135480 167092	mental: 1.6 x 3 = 4.8 x 3 x 3 3.0 1.8 written: 4516 x 37 = 167,092 4516 x 37 31612 135480 167092
Mental/Jottings	**Mental/Jottings**	**Mental/Jottings & Written**	**Mental/Jottings & Written**	**Mental/Jottings & Written**	**Mental/Jottings & Written**

MULTIPLICATION: YEAR 1 OVERVIEW

Number Facts

Consolidate all previous objectives and:

Count in 10s from zero (to the 12th multiple): 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 120, 120.

Count in 2s from zero (to the 12th multiple): 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24.

Count in 5s from zero (to the 12th multiple): 0, 5, 10, 15, 20, 25, 30, 35, 40, 50, 55, 60.

Recall doubles up to 10 + 10, e.g. 8 + 8 = 16 so double 8 = 16

Vocabulary

array, rows and columns

equal groups of …, grouping

odd, even

double, doubling

once, twice, three times… ten times… times as (big, long, wide… and so on)

twice as much/many

Mental Strategies/Jottings

Practical work:

Grouping (equal groups)

e.g. 2 equal groups of 6
and 6 equal groups of 2

Arrays

e.g. 2+2+2+2+2+2+2+2+2+2 = 2 x 7

and 7+7 = 7 x 2

(Ref: Solve one-step problems involving multiplication calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.)

Written Method – end of year expectation

NO FORMAL WRITTEN METHODS IN YEAR 1

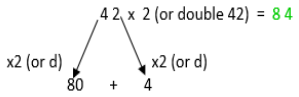
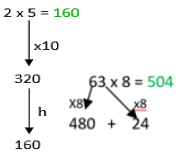
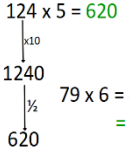
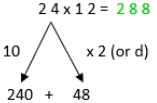
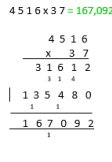
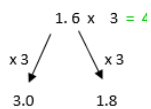
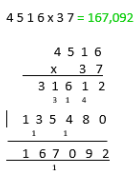
Assessment of Expected Standard

Can pupils count in 2s, 5s and 10s up to the 12th multiple? Can they find the total of several 2p coins? Do they know doubles facts up to double 10 off by heart?

Can pupils use manipulatives or their own pictorial representations to create equal groups and explain what they have shown? e.g. to show how many bicycles there are if there are 14 wheels.

Greater Depth Opportunities

Solve problems, e.g. How many different 'flat' (1 cube deep) cuboids can you build using 12 multilink cubes? Can you record this on squared paper using 2 colours and explain what you have found using numbers?

Reception	Year 2	Year 3	Year 4	Year 5	Year 6
practical only: Represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be grouped equally.	mental only: 	mental:  written: 63 x 8 = 504 63 x 8 504 2	mental:  79 x 6 = 420 + 54 = 474 written: 236 x 7 = 1652 236 x 7 1652 24	mental:  written: 	mental:  written: 
Concrete	Mental/Jottings	Mental/Jottings & Written	Mental/Jottings & Written	Mental/Jottings & Written	Mental/Jottings & Written

MULTIPLICATION: YEAR 2 OVERVIEW

Number Facts

Consolidate all previous objectives and:

Recall multiplication and division **facts for the 10 times table** up to the 12th multiple.

Recall multiplication and division **facts for the 2 times table** up to the 12th multiple.

Recall multiplication and division **facts for the 5 times table** up to the 12th multiple.

Vocabulary

times, multiply, multiplied by

multiple of 2, 5, 10

array, rows and columns

repeated addition

commutative

equal groups of …, grouping

once, twice, three times… ten times… times as (big, long, wide… and so on)

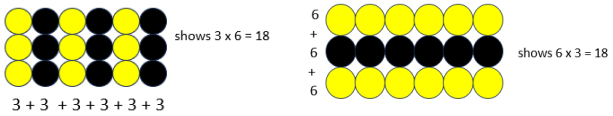
odd, even

double, doubling

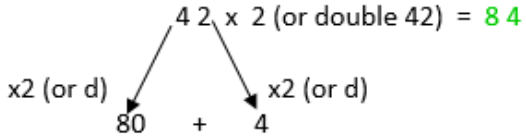
twice as much/many

Mental Strategies/Jottings

Arrays/repeated addition to calculate unknown facts (not 2, 5 or 10 times table)



Double 2-digit numbers



Written Method – end of year expectation

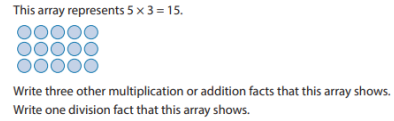
NO FORMAL WRITTEN METHODS IN YEAR 2

Assessment of Expected Standard

EXS: Recall multiplication and division facts for 2, 5 and 10 and use them to solve simple problems, demonstrating an understanding of commutativity as necessary.

Can they instantly recall?

NCETM Teaching for Mastery:



Greater Depth Opportunities

GD: Make deductions outside known multiplication facts.

NCETM Teaching for Mastery:

Reception	Year 1	Year 3	Year 4	Year 5	Year 6
practical only: Represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.	practical and pictorial: Solve problems using concrete objects, pictorial representations, e.g. arrays and equal grouping.	mental: 3 2 x 5 = 160 x10 320 h 160 63 x 8 = 504 x8 x8 480 + 24 written: 63 x 8 = 504 6 3 x 8 5 0 4 2	mental: 124 x 5 = 620 x10 1240 ½ 620 79 x 6 = 420 + 54 = 474 written: 236 x 7 = 1652 2 3 6 x 7 1 6 5 2 2 4	mental: 2 4 x 1 2 = 2 8 8 x 10 x 2 (or d) 240 + 48 written: 4 5 1 6 x 3 7 = 167,092 4 5 1 6 x 3 7 3 1 6 1 2 1 3 5 4 8 0 1 6 7 0 9 2	mental: 1. 6 x 3 = 4 x 3 x 3 3.0 1.8 written: 4 5 1 6 x 3 7 = 167,092 4 5 1 6 x 3 7 3 1 6 1 2 1 3 5 4 8 0 1 6 7 0 9 2
Concrete	**Jottings**	**Mental/Jottings & Written**	**Mental/Jottings & Written**	**Mental/Jottings & Written**	**Mental/Jottings & Written**

MULTIPLICATION: YEAR 3 OVERVIEW

Number Facts :

Consolidate all previous objectives and:

Count in 50s from zero.

0, 50, 100, 150, 200, 250, 300, 350, 400, 450, 500, 550, 600 . . .

Recall multiplication and division **facts for the 3 times table** up to the 12th multiple.

Recall multiplication and division **facts for the 4 times table** up to the 12th multiple.

Recall multiplication and division **facts for the 8 times table** up to the 12th multiple.

Vocabulary

times, multiply, multiplied by

multiple of

product, factor

array, rows and columns

repeated addition

commutative, commutativity

equal groups of …, grouping

once, twice, three times… ten times… times as (big, long, wide… and so on)

double, doubling

twice as much/many

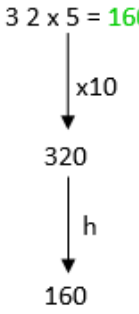
inverse

scaling

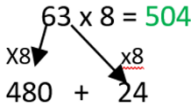
Mental Strategies/Jottings

Consolidate all previous mental strategies and:

Mentally multiply 2-digit and 3-digit numbers by 5



2-digit x 1-digit mental multiplication using times tables they know



63 x 8 = 480 + 24
= 504

Written Method – end of year expectation

2-digit x 1-digit mental multiplication progressing to formal (short multiplication) using times tables they know.

63 x 8 = 504

6 3
x 8
5 0 4
2

16 x 3 = 48

1 6
x 3
4 8
1

Assessment of Expected Standard

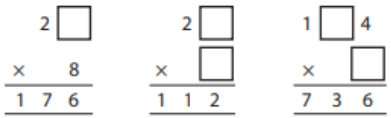
Show different ways to work out each of the following calculations:

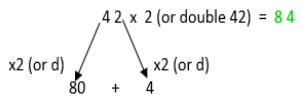
48 x 3 93 x 4 64 x 5 67 x 8

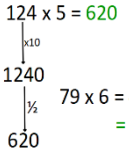
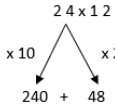
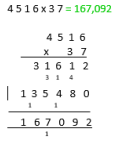
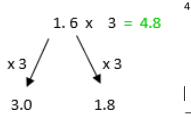
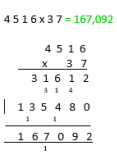
Greater Depth Opportunities

NCETM Teaching for Mastery:

Find the missing digits.



Reception	Year 1	Year 2
practical only: Represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be grouped equally.	practical and pictorial: Solve problems using concrete objects, pictorial representations, e.g. arrays and equal grouping.	mental only: 
Concrete	Jottings	Mental/Jottings

Year 4	Year 5	Year 6
mental:  79 x 6 = 420 + 54 = 474 written: 236 x 7 = 1652 2 3 6 x 7 1 6 5 2 2 4	mental:  written: 	mental:  written: 
Mental/Jottings & Written	Mental/Jottings & Written	Mental/Jottings & Written

MULTIPLICATION: YEAR 4 OVERVIEW

Number Facts

Consolidate all previous objectives and:

Count in 25s.

0, 25, 50, 75, 100, 125, 150, 175, 200, 225, 250, 275, 300 . . .

Recall multiplication and division **facts for the 6 times table** up to the 12th multiple.

Recall multiplication and division **facts for the 11 times table** up to the 12th multiple.

Recall multiplication and division **facts for the 7 times table** up to the 12th multiple.

Recall multiplication and division **facts for the 9 times table** up to the 12th multiple.

Recall multiplication and division **facts for the 12 times table** up to the 12th multiple.

Vocabulary

times, multiply, multiplied by

multiple of

product, factor

array, rows and columns

repeated addition

commutative, commutativity

equal groups of …, grouping

once, twice, three times… ten times…

times as (big, long, wide… and so on)

double, doubling

twice as much/many

scaling

inverse

Mental Strategies/Jottings

Consolidate all previous mental strategies:

Mentally multiply 3-digit numbers by 5

124 x 5 = 620

x10

1240

½

620

2-digit x 1-digit mental multiplication

79 x 6 = 420 + 54

= 474

Written Method – end of year expectation

Formal written method of short multiplication to multiply two-digit and three-digit numbers by a one-digit number, when this cannot be done mentally.

236 x 7 = 1652

```
  2 3 6
x     7
1 6 5 2
  2 4
```

Assessment of Expected Standard

Show different ways to work out each of the following calculations:

315 x 7 36 x 8 246 x 5 67 x 9

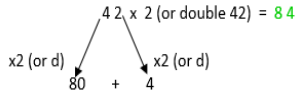
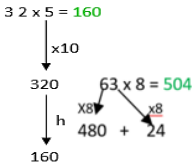
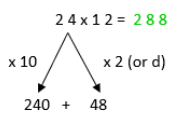
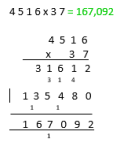
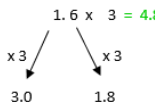
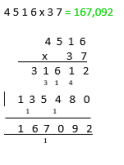
Can they carry out short multiplication accurately?

Greater Depth Opportunities

Can pupils write the number 36 as a product of 3 numbers? Can they do it in different ways?

Can they describe their approach – is it systematic?

Can pupils identify missing digits in a partially completed short multiplication calculation?

Reception	Year 1	Year 2	Year 3	Year 5	Year 6
practical only: Represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be grouped equally.	practical and pictorial: Solve problems using concrete objects, pictorial representations, e.g. arrays and equal grouping.	mental only: 	mental:  written: 63 x 8 = 504 63 x 8 504 2	mental:  written: 	mental:  written: 
Concrete	Jottings	Mental/Jottings	Mental/Jottings & Written	Mental/Jottings & Written	Mental/Jottings & Written

MULTIPLICATION: YEAR 5 OVERVIEW

Number Facts

Consolidate all previous objectives and:

Recall prime numbers up to 19.
2, 3, 5, 7, 11, 13, 17, 19 . . .

Recognise and use square numbers.
1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144.

Find all factor pairs of a number and common factors of two numbers.
e.g. factor pairs of 24 are 1 and 24, 2 and 12, 3 and 8, 4 and 6.
e.g. common factors of 15 and 20 are 1 and 5.

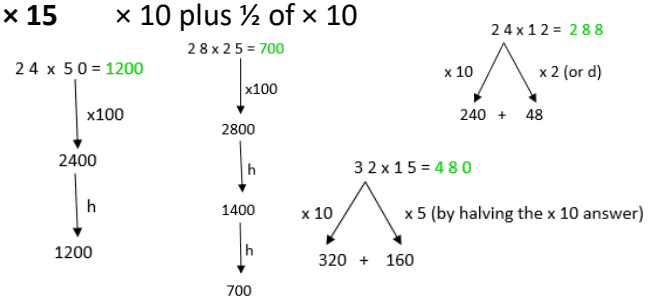
Vocabulary

times, multiply, multiplied by
multiple,
product
factor, factor pairs, common factor
square number, cubed number
commutative, commutativity
equal groups of …, grouping
once, twice, three times… ten times…
times as (big, long, wide… and so on)
double, doubling
scaling
inverse

Mental Strategies/Jottings

Consolidate all previous mental strategies and:

× 50 ½ of × 100 (× 100 then halve it)
× 25 ¼ of × 100 (× 100 then ½ and ½ again)
× 12 × 10 plus × 2 (double)
× 15 × 10 plus ½ of × 10



Written Method – end of year expectation

Formal method of short multiplication for multiplication of up to 4-digit by 1-digit numbers when this cannot be done mentally.

1346 x 7 = 9422

```
  1346
 x    7
 9422
 2 3 4
```

Formal method of long multiplication for multiplication of up to 4-digit by 2-digit numbers.

4516 x 37 = 167,092

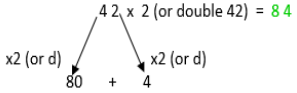
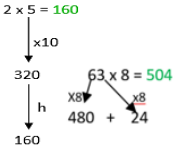
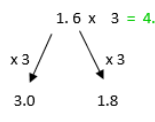
```
   4 5 1 6
 x   3 7
 3 1 6 1 2
 3 1 4
 1 3 5 4 8 0
 1     1
 1 6 7 0 9 2
 1
```

Assessment of Expected Standard

Can pupils recite the prime numbers up to 19 and square numbers up to 144? Can they explain why 1 is not prime? (It does not have 2 factors)
Do pupils spot when a mental strategy is more efficient than a written strategy?
Can they carry out short and long multiplication accurately and identify missing digits in a partially completed short multiplication calculation?

Greater Depth Opportunities

Can pupils identify missing digits in a partially completed long multiplication calculation?
Can they diagnose the errors in an incorrect long multiplication calculation and give advice on how to improve?

Reception	Year 1	Year 2	Year 3	Year 4	Year 6
practical only: Represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be grouped equally.	practical and pictorial: Solve problems using concrete objects, pictorial representations, e.g. arrays and equal grouping.	mental only: 	mental:  written: 63 x 8 = 504 63 x 8 504 2	mental: 124 x 5 = 620 x10 1240 ½ 620 79 x 6 = 420 + 54 = 474 written: 236 x 7 = 1652 236 x 7 1652 2 4	mental:  written: 4516 x 37 = 167,092 4516 x 37 31612 135480 167092
Concrete	**Jottings**	**Mental/Jottings**	**Mental/Jottings & Written**	**Mental/Jottings & Written**	**Mental/Jottings & Written**

MULTIPLICATION: YEAR 6 OVERVIEW

Number Facts

Consolidate all previous objectives and:

Recall equivalences between simple fractions, decimals and percentages.

Convert measurements (length, mass, volume and time) **from smaller units of measure to larger and vice-versa.**

Fraction	Decimal	Percentage
1/10	0.1	10%
2/10 = 1/5	0.2	20%
¼	0.25	25%
3/10	0.3	30%
4/10 = 2/5	0.4	40%
5/10 = ½	0.5	50%
6/10 = 3/5	0.6	60%
7/10	0.7	70%
¾	0.75	75%
8/10 = 4/5	0.8	80%
9/10	0.9	90%
10/10 = 1	1.0	100%

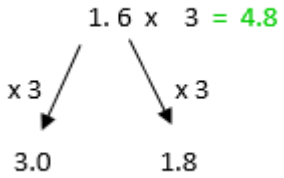
Vocabulary

times, multiply, multiplied by
multiple, common multiple
product
factor, factor pairs, common factor
square number, cubed number
commutative, commutativity
equal groups of …, grouping
once, twice, three times… ten times… times as (big, long, wide… and so on)
double, doubling
scaling
inverse

Mental Strategies/Jottings

Consolidate all previous mental strategies, including Year 5 strategies for x50, x25, x12, x15

Mentally multiply decimal numbers with one decimal place.



Written Method – end of year expectation

Continue formal method of short multiplication (as Year 5) for multiplication of up to 4-digit by 1-digit numbers when this cannot be done mentally.

1346 x 7 = 9422

1346 x 7 = 9422
2 3 4

2.4 x 3 = 7.2

2.4 x 3 = 7.2
1

Formal method of long multiplication for multiplication of 4-digit by 2-digit numbers.

4516 x 37 = 167,092

4516 x 37 = 167,092
3 1 4 2
3 1 4
1 3 5 4 8 0
1 1
1 6 7 0 9 2
1

Assessment of Expected Standard

Do pupils recall equivalent fractions, decimals and percentages?
Do pupils spot when a mental strategy is more efficient than a written strategy?
Can pupils identify missing digits in a partially completed long multiplication calculation?
Can they explain why long multiplication is not used to multiply a number by a multiple of 10, e .g. 272 x 30

Greater Depth Opportunities

NCETM Teaching for Mastery:

In each pair of calculations, which one would you prefer to work out?

- (a) 35 × 0·3 + 35 × 0·7 **or** (b) 3·5 × 0·3 + 35 × 7
- (c) 6·4 × 1·27 – 64 × 0·1 **or** (d) 6·4 × 1·27 – 64 × 0·027

Explain your choices.

Reception	Year 1	Year 2	Year 3	Year 4	Year 5
practical only: Represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be grouped equally.	practical and pictorial: Solve problems using concrete objects, pictorial representations, e.g. arrays and equal grouping.	mental only: 42 x 2 (or double 42) = 84 x2 (or d) x2 (or d) 80 + 4	mental: 32 x 5 = 160 x10 320 h 160 63 x 8 = 504 x8 x8 480 + 24 written: 63 x 8 = 504 63 x 8 504 2	mental: 124 x 5 = 620 x10 1240 ½ 620 79 x 6 = 420 + 54 = 474 written: 236 x 7 = 1652 236 x 7 1652 2 4	mental: 24 x 12 = 288 x 10 x 2 (or d) 240 + 48 written: 4516 x 37 = 167,092 4516 x 37 31612 3 1 4 135480 1 1 167092 1
Concrete	**Jottings**	**Mental/Jottings**	**Mental/Jottings & Written**	**Mental/Jottings & Written**	**Mental/Jottings & Written**

DIVISION: RECEPTION OVERVIEW

Number Facts

Automatically recall (without reference to rhymes, counting or other aids) . . . **double facts, extending to the corresponding halving facts** at least up to half of 10 is 5, e.g. half of 8 is 4.

Doubling Facts to 10					
0 + 0 = 0	1 + 1 = 2	2 + 2 = 4	3 + 3 = 6	4 + 4 = 8	5 + 5 = 10
double 0 is 0	double 1 is 2	double 2 is 4	double 3 is 6	double 4 is 8	double 5 is 10

Extend to **Halving Facts Within 10**				
half of 2 is 1	half of 4 is 2	half of 6 is 3	half of 8 is 4	half of 10 is 5

Vocabulary

equal groups of …

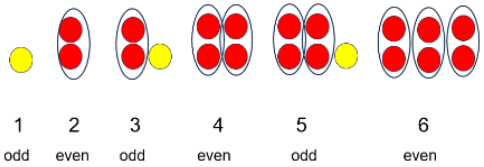
share equally, sharing between

half, halve

odd, even

Mental Strategies/Jottings

Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally, e.g. odds and evens 1, 2, 3, 4, 5, 6



Written Method – end of year expectation

NO FORMAL WRITTEN METHODS IN EYFS

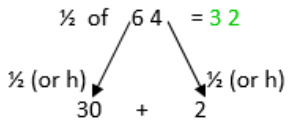
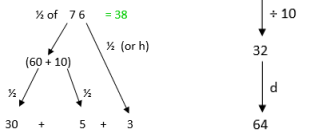
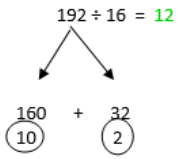
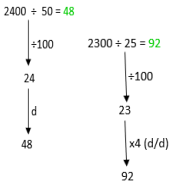
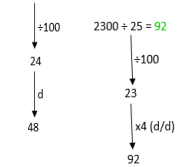
Assessment of Expected Standard

Can pupils use manipulatives to show and talk about a pattern of doubles and halves, e.g. Cuisenaire Rods, multilink cubes in 2 colours?

Can they recall halving facts up to 5 + 5?

Greater Depth Opportunities

Can pupils recognise when they *can't* share a quantity equally between two because they know it is an odd number?
Can they use manipulatives and pictorial representations to explain this?

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
practical and pictorial: Mental, simple jottings or own pictorial representations e.g. sharing equally to solve a real-life problem.	mental only:  	mental (or with efficient jottings) only:  	mental (or with efficient jottings) only:  	mental:  written: 1647 ÷ 3 = 549 0549 3 1 6 1 4 2 7	mental: (as Year 5)  written: 1440 ÷ 32 = 45 0045 32 1440 -128↓. 160 -160 0
Mental/Jottings	**Mental/Jottings**	**Mental/Jottings & Written**	**Mental/Jottings & Written**	**Mental/Jottings & Written**	**Mental/Jottings & Written**

DIVISION: YEAR 1 OVERVIEW

Number Facts

Consolidate all previous objectives and:

Recall halves of even numbers up to half of 20, e.g. half of 18 is 9.

Halving Facts Within 20				
half of 2 is 1	half of 4 is 2	half of 6 is 3	half of 8 is 4	half of 10 is 5
half of 12 is 6	half of 14 is 7	half of 16 is 8	half of 18 is 9	half of 20 is 10

Vocabulary

array, rows and columns

repeated subtraction

equal groups of …

share equally, sharing between

odd, even

half, halve, half of.., quarter of..

Mental Strategies/Jottings

Practical work:

Sharing, e.g. share 18 between 2

Grouping (equal groups), e.g. 18 grouped in 2s

and 18 grouped in 9s

Linked to **Arrays**, e.g. 2 x 9 and 9 x 2 (rows and columns)

(Ref: Solve one-step problems involving . . . division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.)

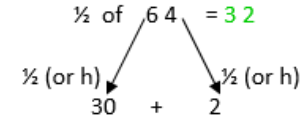
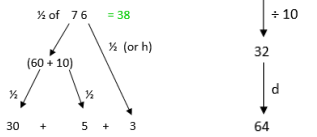
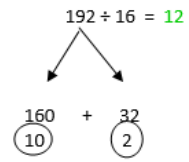
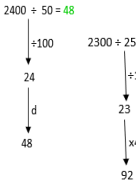
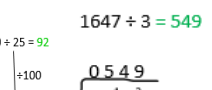
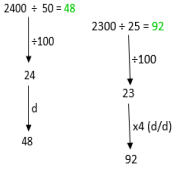
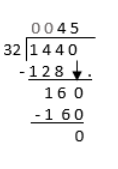
Written Method – end of year expectation

NO FORMAL WRITTEN METHODS IN YEAR 1

Assessment of Expected Standard

Greater Depth Opportunities

Can pupils solve problems? e.g. If you had 20 sweets, how many different ways could you arrange them into party bags with an equal amount in each bag? How many bags do you use each time? Can you show this pictorially?

Reception	Year 2	Year 3	Year 4	Year 5	Year 6
practical only: Represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be shared equally.	mental only: 	mental (or with efficient jottings) only: 	mental (or with efficient jottings) only: 	mental:  written: 	mental: (as Year 5)  written: 
Concrete	**Mental/Jottings**	**Mental/Jottings & Written**	**Mental/Jottings & Written**	**Mental/Jottings & Written**	**Mental/Jottings & Written**

DIVISION: YEAR 2 OVERVIEW

Number Facts

Consolidate all previous objectives and:

Recall multiplication and **division facts for the 10 times table** up to the 12th multiple.

Recall multiplication and **division facts for the 2 times table** up to the 12th multiple.

Recall multiplication and **division facts for the 5 times table** up to the 12th multiple.

Vocabulary

array, rows and columns

repeated subtraction

equal groups of …

share equally, sharing between

odd, even

half, halving, half of.., quarter of..

divide, divided by, divided into

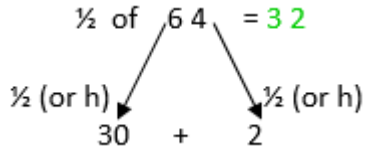
dividend, divisor, quotient

Mental Strategies/Jottings

Sharing 15 ÷ 3

Grouping 15 ÷ 3

Halve 2-digit even numbers.



Written Method – end of year expectation

NO FORMAL WRITTEN METHODS IN YEAR 2

Assessment of Expected Standard

EXS: Recall multiplication and division facts for 2, 5 and 10 and use them to solve simple problems, demonstrating an understanding of commutativity as necessary.

Can they *instantly* recall facts? e.g. what number divided by 5 is 7?

NCETM Teaching for Mastery:

Two friends share 12 sweets equally between them. How many do they each get?
Write this as a division number sentence.

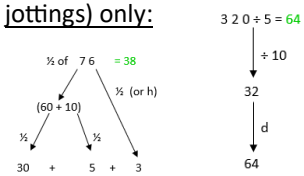
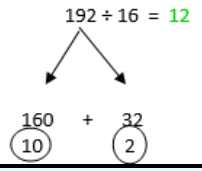
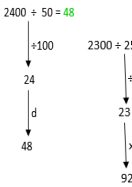
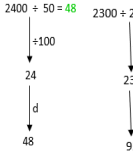
Make up two more sharing stories like this one.

Greater Depth Opportunities

NCETM Teaching for Mastery:

Two friends want to buy some marbles and then share them out equally between them.
They could buy a bag of 13 marbles, a bag of 14 marbles or a bag of 19 marbles.
What size bag should they buy so that they can share them equally?

What other numbers of marbles could be shared equally?

Reception	Year 1	Year 3	Year 4	Year 5	Year 6
practical only: Represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be shared equally.	practical and pictorial: Mental, simple jottings or own pictorial representations e.g. sharing equally to solve a real-life problem.	mental (or with efficient jottings) only: 	mental (or with efficient jottings) only: 	mental:  written: 1647 ÷ 3 = 549 0549 3 1 6 14 27	mental: (as Year 5)  written: 1440 ÷ 32 = 45 0045 32 1440 -128 160 -160 0
Concrete	Jottings	Mental/Jottings & Written	Mental/Jottings & Written	Mental/Jottings & Written	Mental/Jottings & Written

DIVISION: YEAR 3 OVERVIEW

Number Facts

Consolidate all previous objectives and:

Recall multiplication and **division facts for the 3 times table** up to the 12th multiple.

Recall multiplication and **division facts for the 4 times table** up to the 12th multiple.

Recall multiplication and **division facts for the 8 times table** up to the 12th multiple.

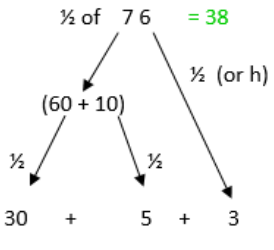
Vocabulary

array, rows and columns	dividend, divisor, quotient
repeated subtraction	half, halving
equal groups of …	remainder
share equally, sharing between	inverse
divide, divided by, divided into	factor

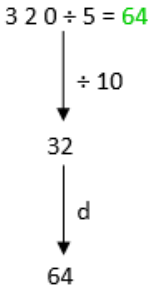
Mental Strategies/Jottings

Consolidate Year 2 mental strategies and:

Halve 2-digit and 3-digit even numbers:



Divide 3-digit multiples of 10 by 5:



Written Method – end of year expectation

Use known division facts to solve problems in context (2, 5, 10, 3, 4 and 8 times tables).

NO FORMAL WRITTEN METHODS IN YEAR 3

Assessment of Expected Standard

Can pupils *instantly* recall division facts? e.g. what number divided by 3 is 9?

Can they use jottings or mental strategies to halve 2-digit numbers or to divide 3-digit multiples of 10 by 5?

NCETM Teaching For Mastery:

The following problems can be solved by using the calculation 8 ÷ 2. True or false?

- There are 2 bags of bread rolls that have 8 rolls in each bag. How many rolls are there altogether?
- A boat holds 2 people. How many boats are needed for 8 people?
- I have 8 pencils and give 2 pencils to each person. How many people receive pencils?
- I have 8 pencils and give 2 away. How many do I have left?

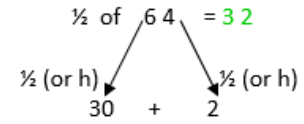
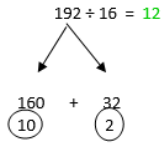
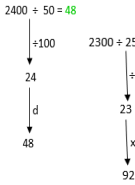
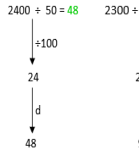
Greater Depth Opportunities

Can pupils mentally solve problems involving remainders using known division facts?

And *NCETM Teaching for Mastery:*

What is the relationship between these calculations?

2 × 3	4 × 3
2 × 30	4 × 30
20 × 3	40 × 3
20 × 3 × 10	40 × 3 × 10

Reception	Year 1	Year 2	Year 4	Year 5	Year 6
practical only: Represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be shared equally.	practical and pictorial: Mental, simple jottings or own pictorial representations e.g. sharing equally to solve a real-life problem.	mental only: 	mental (or with efficient jottings) only: 	mental:  written: 1647 ÷ 3 = 549 0549 3 1 6 14 27	mental: (as Year 5)  written: 1440 ÷ 32 = 45 0045 32 1440 -128 160 -160 0
Concrete	**Jottings**	**Mental/Jottings**	**Mental/Jottings & Written**	**Mental/Jottings & Written**	**Mental/Jottings & Written**

DIVISION: YEAR 4 OVERVIEW

Number Facts

Consolidate all previous objectives and:

Recall multiplication and **division facts for the 6 times table** up to the 12th multiple.

Recall multiplication and **division facts for the 11 times table** up to the 12th multiple.

Recall multiplication and **division facts for the 7 times table** up to the 12th multiple.

Recall multiplication and **division facts for the 9 times table** up to the 12th multiple.

Recall multiplication and **division facts for the 12 times table** up to the 12th multiple.

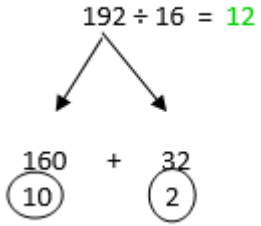
Vocabulary

repeated subtraction
equal groups of …
share equally, sharing between
divide, divided by, divided into
dividend, divisor, quotient
half, halving
remainder
inverse
factor

Mental Strategies/Jottings

Consolidate Years 2 and 3 mental strategies and:

Mentally divide 2- or 3-digit numbers by 1- or 2-digit numbers using times tables beyond the 12th multiple, where there are no remainders.



Written Method – end of year expectation

Use known division facts to solve problems in context (times tables up to 12 x 12).

NO FORMAL WRITTEN METHODS IN YEAR 4

Assessment of Expected Standard

Can pupils *instantly* recall division facts? e.g. what number divided by 7 is 8?

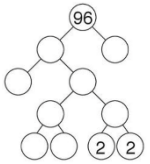
Can pupils solve problems using known division facts, including involving remainders?

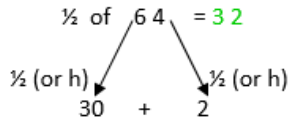
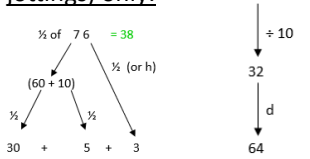
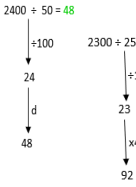
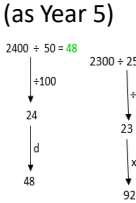
Can they use a jotting or mental strategy to divide beyond the 12th multiple of the divisor (but less than the 12th)?

Greater Depth Opportunities

Can pupils solve factor tree puzzles in different ways and create their own to challenge others?

Can they show resilience and solve *A First Product Sudoku* from *NRich?*



Reception	Year 1	Year 2	Year 3	Year 5	Year 6
practical only: Represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be shared equally.	practical and pictorial: Mental, simple jottings or own pictorial representations e.g. sharing equally to solve a real-life problem.	mental only: 	mental (or with efficient jottings) only: 	mental:  written: 1647 ÷ 3 = 549 0549 3 1 6 1 4 2 7	mental: (as Year 5)  written: 1440 ÷ 32 = 45 0045 32 1440 -128 160 -160 0
Concrete	Jottings	Mental/Jottings	Mental/Jottings & Written	Mental/Jottings & Written	Mental/Jottings & Written

DIVISION: YEAR 5 OVERVIEW

Number Facts

Consolidate all previous objectives and:

Recognise and use square numbers, including for example (square roots), *what number squared is 144?*
1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144.

Find all factor pairs of a number and common factors of two numbers.
e.g. factor pairs of 24 are 1 and 24, 2 and 12, 3 and 8, 4 and 6.
e.g. common factors of 15 and 20 are 1 and 5.

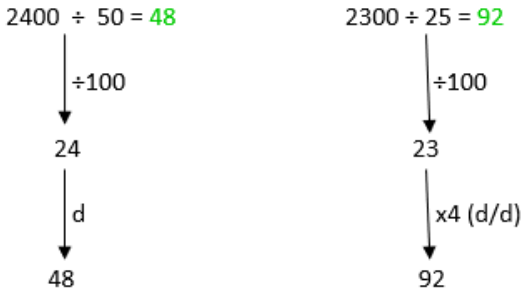
Vocabulary

repeated subtraction
equal groups of …
share equally, sharing between
divide, divided by, divided into
dividend, divisor, quotient
half, halving
remainder
inverse
factor, factor pairs, common factor
prime number, composite number

Mental Strategies/Jottings

Consolidate all previous mental strategies and:

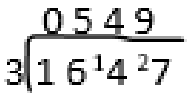
Divide by 50 and 25 with whole number answers:



Written Method – end of year expectation

Formal method of short division for division of numbers up to 4-digits by a one-digit number and interpret remainders appropriately for the context.

1647 ÷ 3 = 549



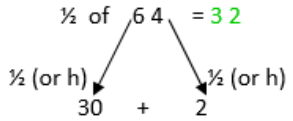
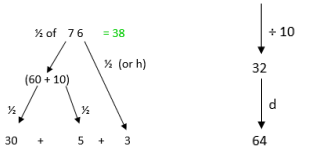
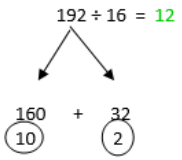
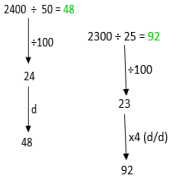
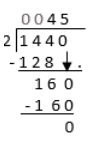
Assessment of Expected Standard

Do pupils spot when a mental strategy is more efficient than a written strategy?
Can pupils carry out short division accurately, using known division facts and remainders to solve a problem?

Greater Depth Opportunities

NCETM Teaching for Mastery :

Fill in the missing numbers:
□ ÷120 = 117 ÷ 13 = 10 800 ÷ □ = 234 ÷ □

Reception	Year 1	Year 2	Year 3	Year 4	Year 6
practical only: Represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be shared equally.	practical and pictorial: Mental, simple jottings or own pictorial representations e.g. sharing equally to solve a real-life problem.	mental only: 	mental (or with efficient jottings) only: 	mental (or with efficient jottings) only: 	mental: (as Year 5)  written: 1440 ÷ 32 = 45 
Concrete	**Jottings**	**Mental/Jottings**	**Mental/Jottings & Written**	**Mental/Jottings & Written**	**Mental/Jottings & Written**

DIVISION: YEAR 6 OVERVIEW

Number Facts

Consolidate all previous objectives and:

Recall equivalences between simple fractions, decimals and percentages.

Convert measurements (length, mass, volume and time) **from smaller units of measure to larger and vice-versa.**

Fraction	Decimal	Percentage
1/10	0.1	10%
2/10 = 1/5	0.2	20%
¼	0.25	25%
3/10	0.3	30%
4/10 = 2/5	0.4	40%
5/10 = ½	0.5	50%
6/10 = 3/5	0.6	60%
7/10	0.7	70%
¾	0.75	75%
8/10 = 4/5	0.8	80%
9/10	0.9	90%
10/10 = 1	1.0	100%

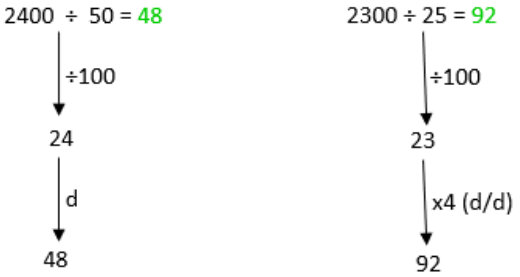
Vocabulary

repeated subtraction
equal groups of …
share equally, sharing between
divide, divided by, divided into
dividend, divisor, quotient
half, halving
remainder
inverse
factor, common factor, factor pairs
common multiple
prime number, composite number

Mental Strategies/Jottings

Consolidate all previous mental strategies, including:

Year 5: Divide by 50 and 25 with whole number answers:



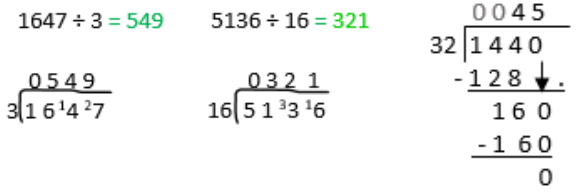
Written Method – end of year expectation

Formal method of short division and long division for division of numbers up to 4-digits by a 2-digit whole number and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

1440 ÷ 32 = 45

1647 ÷ 3 = 549

5136 ÷ 16 = 321



Assessment of Expected Standard

Do pupils spot when a mental strategy is more efficient than a written strategy?
Do pupils know and understand the difference between short and long division and know when to use each method? Can they use both methods accurately?

Greater Depth Opportunities

Can pupils identify missing digits in a partially completed short division calculation?

And *NCETM Teaching for Mastery:*

In each pair of calculations, which one would you prefer to work out?

- (e) 52·4 ÷ 0·7 + 524 ÷ 7 **or** (f) 52·4 ÷ 0·7 – 524 ÷ 7
- (g) 31·2 ÷ 3 – 2·4 ÷ 6 **or** (h) 31·2 ÷ 3 – 1·2 ÷ 0·3

Explain your choices.

Reception	Year 1	Year 2	Year 3	Year 4	Year 5
practical only: Represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be shared equally.	practical and pictorial: Mental, simple jottings or own pictorial representations e.g. sharing equally to solve a real-life problem.	mental only: ½ of 64 = 32 ½ (or h) 30 + ½ (or h) 2	mental (or with efficient jottings) only: ½ of 76 = 38 (60 + 10) ½ (or h) ½ ½ 30 + 5 + 3 320 ÷ 5 = 64 ÷ 10 32 d 64	mental (or with efficient jottings) only: 192 ÷ 16 = 12 160 + 32 10 2	mental: 2400 ÷ 50 = 48 ÷100 24 d 48 2300 ÷ 25 = 92 ÷100 23 x4 (d/d) 92 written: 1647 ÷ 3 = 549 0549 3 1 6 14 27
Concrete	**Jottings**	**Mental/Jottings**	**Mental/Jottings & Written**	**Mental/Jottings & Written**	**Mental/Jottings & Written**